Star-Image Centroiding And Attitude Determination for Star Trackers

YOON. H C

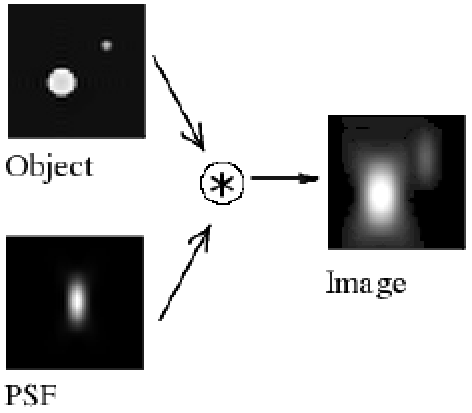
CONTENT

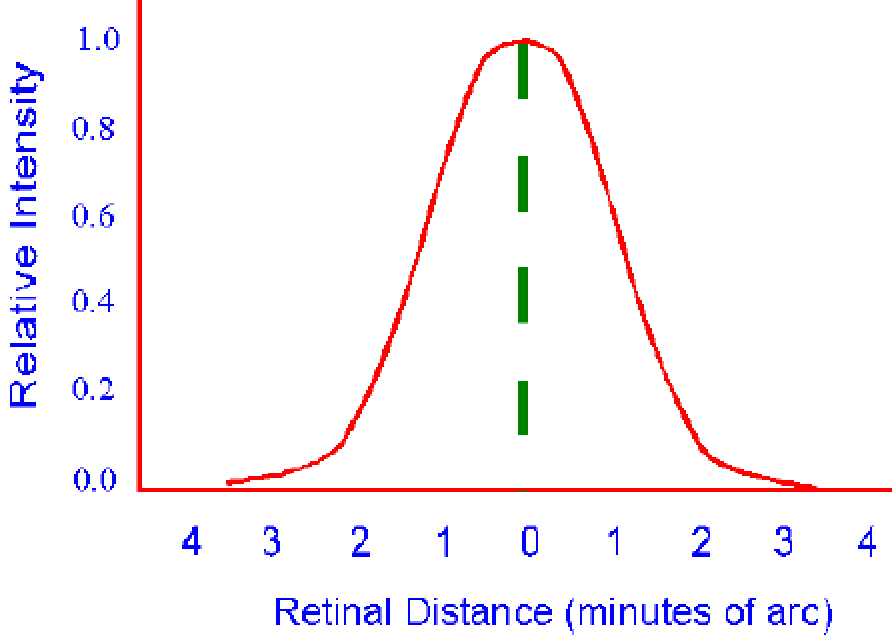
Electro – Optical Modeling And Image Simulator Design

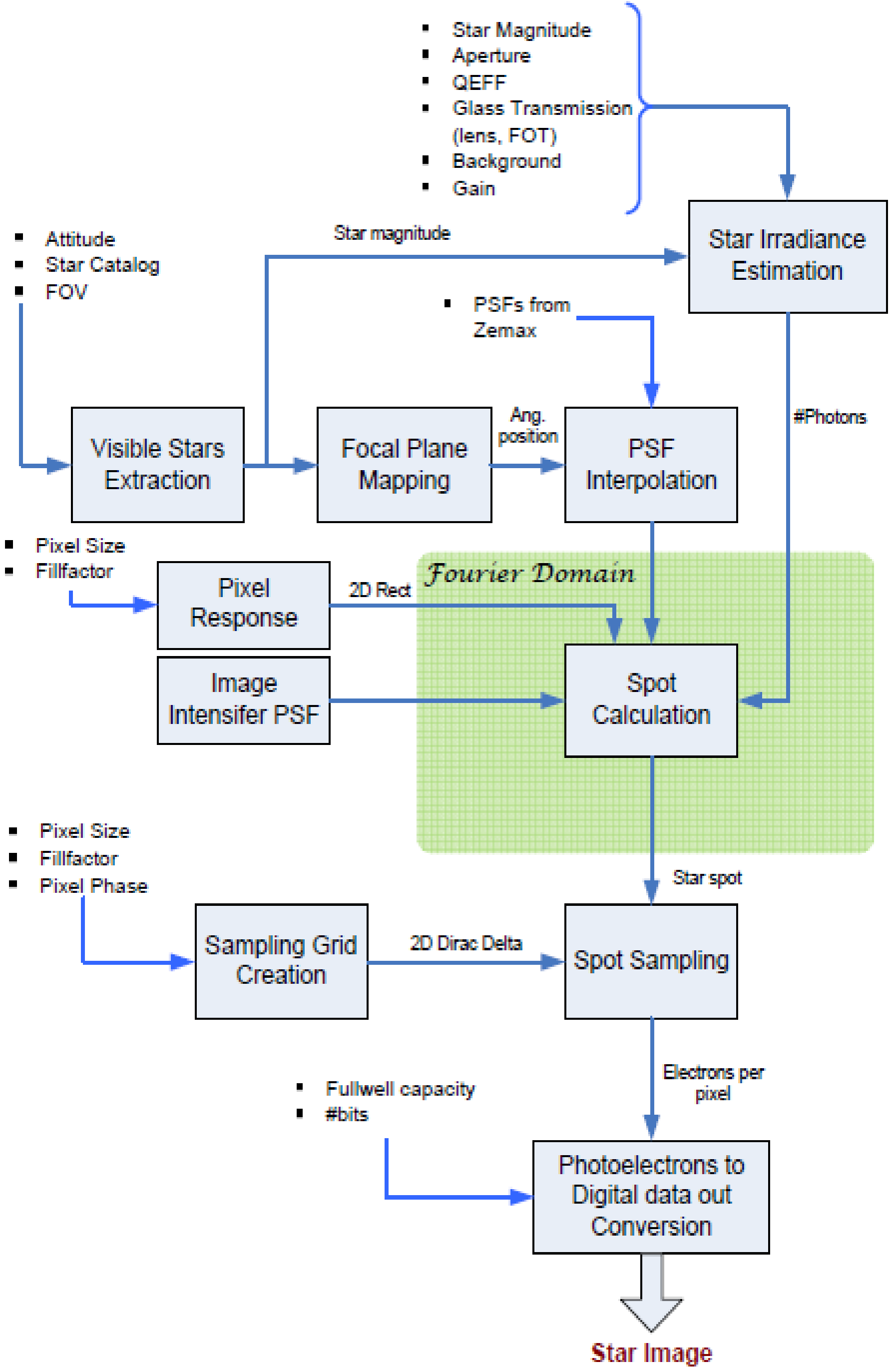
Star Position Estimation Improvements

Simulation

Electro – Optical Modeling And Image Simulator Design







- Star Magnitude
- Aperture
- QEFF
- Glass Transmission (lens, FOT)
- Background
- Gain

- Attitude
- Star Catalog
- FOV

Star magnitude

Star Irradiance Estimation

- PSFs from Zemax

#Photons

Visible Stars Extraction

Focal Plane Mapping

Ang. position

PSF Interpolation

- Pixel Size
- Fillfactor

Fourier Domain

Pixel Response

2D Rect

Image Intensifer PSF

Spot Calculation

- Pixel Size
- Fillfactor
- Pixel Phase

Star spot

Sampling Grid Creation

2D Dirac Delta

Spot Sampling

- Fullwell capacity
- #bits

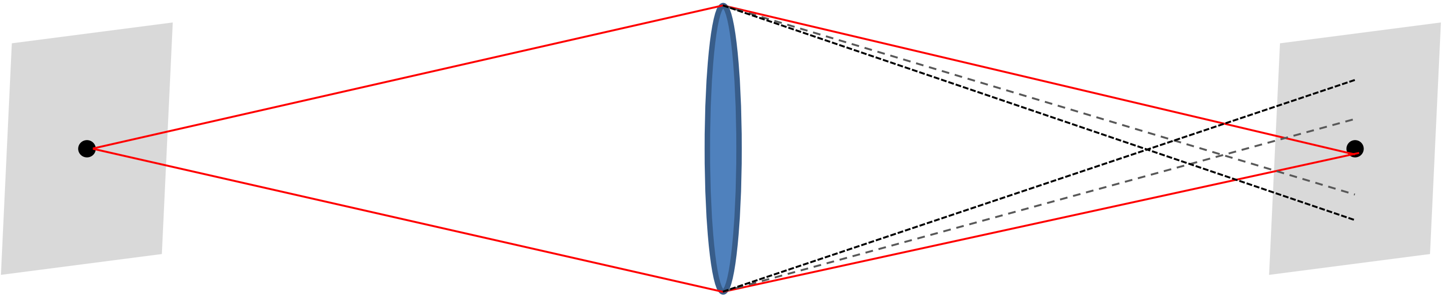
Electrons per pixel

Photoelectrons to Digital data out Conversion

Star Image

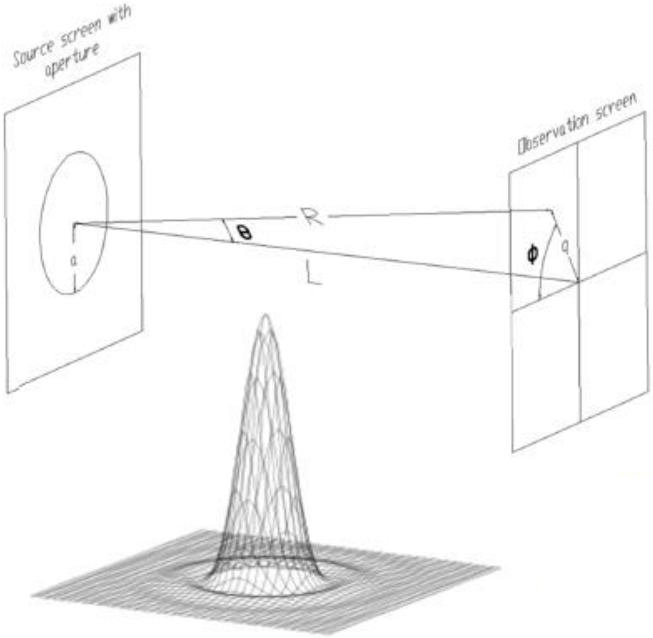
Electro – Optical Modeling
And
Image Simulator Design

Point Spread Function (PSF)



Light point

Spread



-빛의 확산인 빛의 분포 정도를 Point Spread Function(PSF) 라고 부른다.

Electro – Optical Modeling And Image Simulator Design

- 2D Gaussian function

$$f(x,y)=\frac{E}{(2\pi\sigma_x\sigma_y)}e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}}e^{-\frac{(y-y_0)^2}{2\sigma_y{}^2}}$$

$$where\,E,\ \square t\times e^{\frac{(m_0-m)}{2.5}} \longrightarrow \text{Star energy}$$

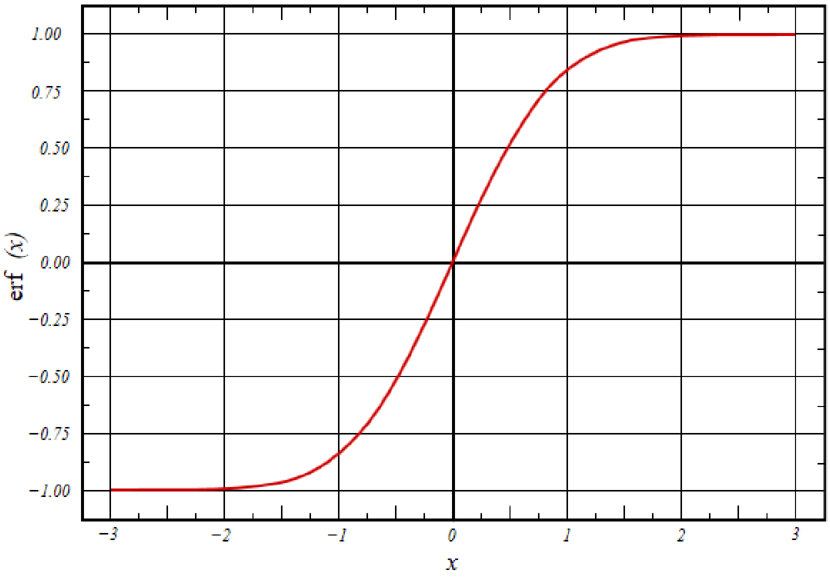
- Error function (ERF)

Electro – Optical Modeling And Image Simulator Design

- Error function (ERF)

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

$$\int_0^\infty e^{-\frac{(x-x_0)^2}{2\sigma_x^2}}\,dt = \frac{2}{\sqrt{\pi}}\left[1-erf\left(\frac{x_0-h}{\sigma_x\sqrt{2}}\right)\right]\sigma_x$$



Electro – Optical Modeling And Image Simulator Design

$$f(x,y)=\frac{E}{(2\pi\sigma_x\sigma_y)}\boxed{e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}}}e^{-\frac{(y-y_0)^2}{2\sigma_y{}^2}}$$

$$\int_{x_1}^{x_2} e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}}dx=\int_{x_1}^{\infty} e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}}dx-\int_{x2}^{\infty} e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}}dx$$

$$=\sqrt{\frac{\pi}{2}}\left[\varepsilon\left(x_2\right)-\varepsilon\left(x_1\right)\right]\sigma_x$$

$$where\ \varepsilon(x_1),\ erf\left(\frac{x_0-x_1}{\sigma_x\sqrt{2}}\right) \qquad \varepsilon(x_2),\ erf\left(\frac{x_0-x_2}{\sigma_x\sqrt{2}}\right)$$

Electro – Optical Modeling
And
Image Simulator Design

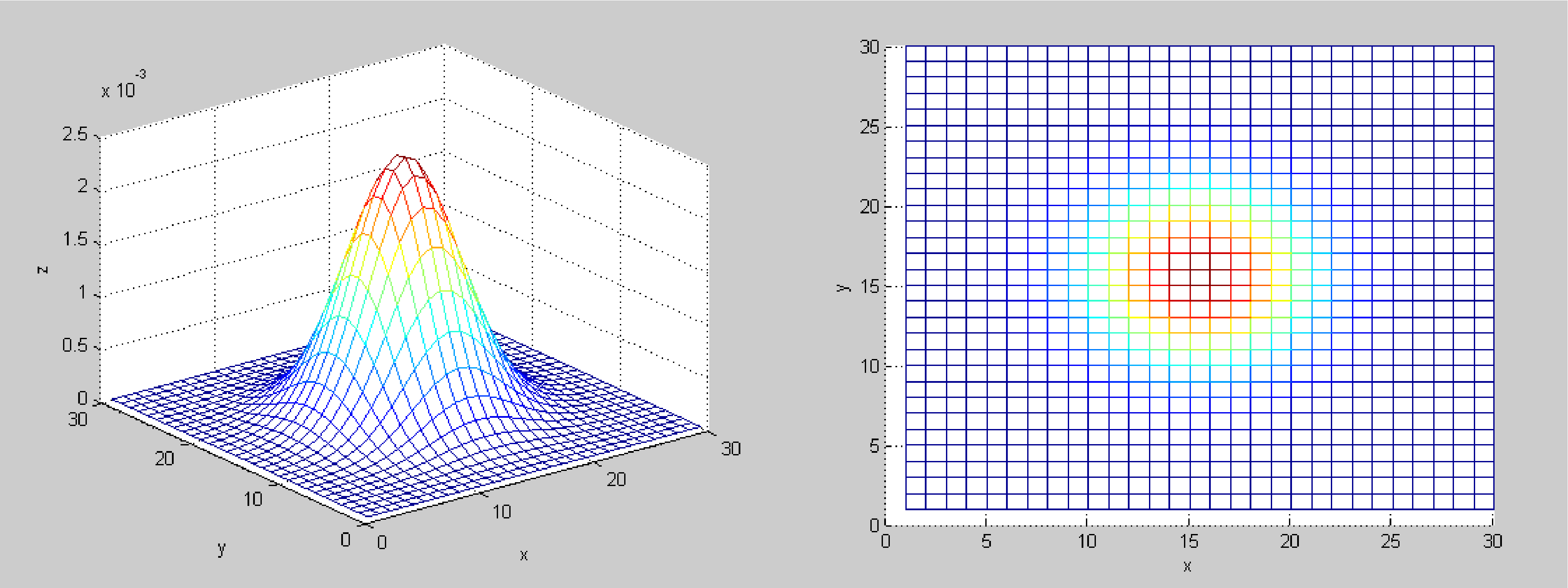
$$f(x,y) = \frac{E}{(2\pi\sigma_x\sigma_y)} e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}} e^{-\frac{(y-y_0)^2}{2\sigma_y{}^2}}$$

$$\Rightarrow \frac{E}{(2\pi\sigma_x\sigma_y)} \left[\int_{y_1}^{y_2} e^{-\frac{(y-y_0)^2}{2\sigma_y{}^2}} dy \right] \left[\int_{x_1}^{x_2} e^{-\frac{(x-x_0)^2}{2\sigma_x{}^2}} dx \right]$$

$$\Rightarrow \frac{E}{4} [\varepsilon(y_2) - \varepsilon(y_1)] [\varepsilon(x_2) - \varepsilon(x_1)]$$

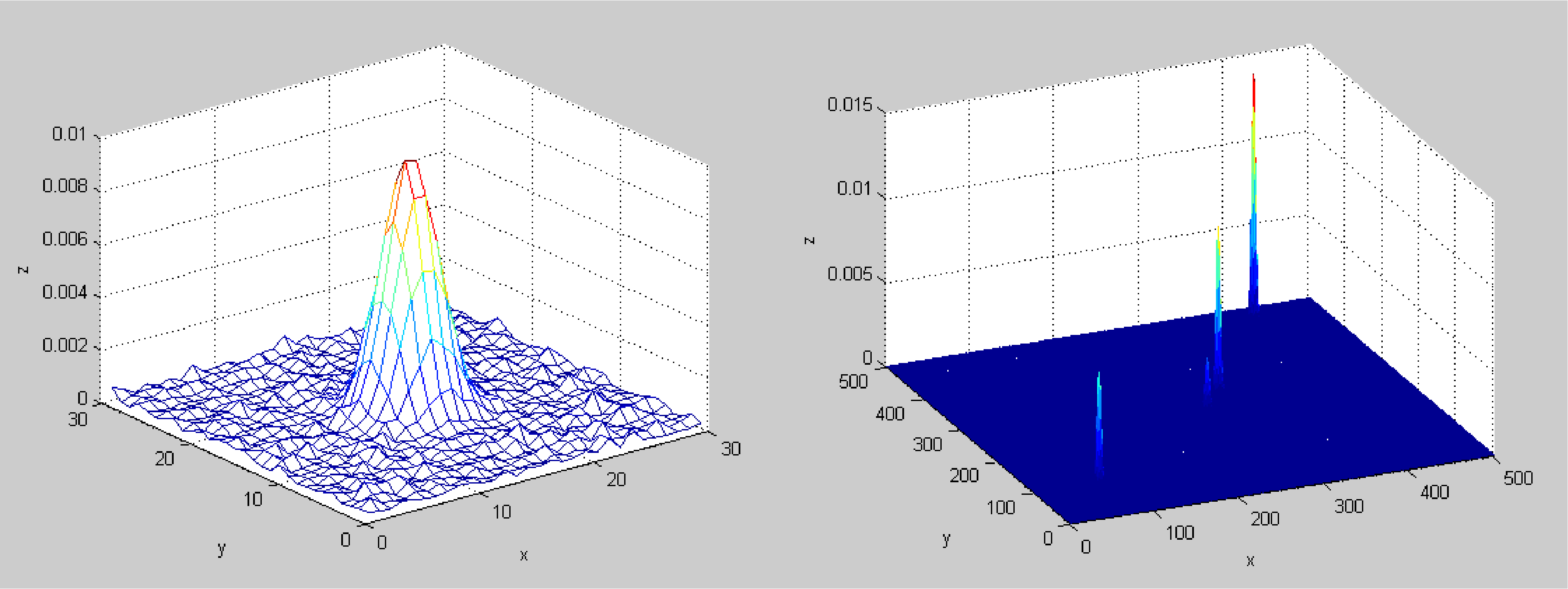
Electro – Optical Modeling And Image Simulator Design

- Simulation



Electro – Optical Modeling And Image Simulator Design

- Simulation



Star Position
Estimation Improvements

- Methods

Center Of Mass

Non-Linear Gaussian Fitting

Linear Least Squares Log of Gaussian Estimation

$$x_{CM} = \frac{\sum_{i,j} x_{ij} E_{ij}}{\sum_{i,j} E_{ij}}$$

$$\Delta_{k+1} = [F_k^{\mathrm{T}} F_k]^{-1} F_k^{\mathrm{T}} R_k$$

$$z = (A^T W A)^{-1} A^T W F$$

Star Position Estimation Improvements

- Center Of Mass(COM)

$$x_c = x_M + \frac{\displaystyle\sum_{i=-n/2}^{n/2}\sum_{j=-m/2}^{m/2}\left[P\left(x_M - i,\ y_M - j\right) - B\right] i}{\displaystyle\sum_{i=-n/2}^{n/2}\sum_{j=-m/2}^{m/2}\left[P\left(x_M - i,\ y_M - j\right) - B\right]}$$

$$y_c = y_M + \frac{\displaystyle\sum_{i=-n/2}^{n/2}\sum_{j=-m/2}^{m/2}\left[P\left(x_M - i,\ y_M - j\right) - B\right] j}{\displaystyle\sum_{i=-n/2}^{n/2}\sum_{j=-m/2}^{m/2}\left[P\left(x_M - i,\ y_M - j\right) - B\right]}$$

$(x_M\ y_M)$: location of the peak of the intensity distribution

n, m : mask sizes along x and y directions

B : Background noise intensity (constant)

Star Position
Estimation Improvements

- Non-Linear 2-D Gaussian Fitting

$$P(x,y) = \frac{t_{int}}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{m_0 - m}{2.5}\right)} e^{-\frac{(x-x_c)^2}{2\sigma_x^2}} e^{-\frac{(y-y_c)^2}{2\sigma_y^2}}$$

$$\Rightarrow \quad \ln P(x,y) = \underbrace{\ln \frac{t_{int}}{2\pi\sigma_x\sigma_y} - \frac{m_0 - m}{2.5}}_{z_c} - \frac{(x-x_c)^2}{2\sigma_x^2} - \frac{(y-y_c)^2}{2\sigma_y^2}$$

$$\Rightarrow \quad z_i = z_c - -\frac{(x_i-x_c)^2}{2\sigma_x^2} - \frac{(y_i-y_c)^2}{2\sigma_y^2} = f\left(x_c,\, y_c,\, z_c\right)$$

$$\epsilon_i = z_i - f\left(x_c,\, y_c,\, z_c\right) \qquad L = \textstyle\sum_{i=1}^{n} \alpha_i \epsilon_i^2 \quad \textit{where } \alpha, \textit{relative weights}$$

Star Position
Estimation Improvements

- Non-Linear 2-D Gaussian Fitting

$$\frac{\partial L}{\partial x_0}=0,\quad \frac{\partial L}{\partial y_0}=0,\quad \frac{\partial L}{\partial z_0}=0$$

⇨

$$\begin{aligned}
\frac{\partial L}{\partial x_c}&=0 \Rightarrow \sum_i \left[2\left(z_i-z_c\right)+\sigma_x^{-2}\left(x_i-x_c\right)^2+\sigma_y^{-2}\left(y_i-y_c\right)^2\right]\left(x_i-x_c\right)=0\\
\frac{\partial L}{\partial y_c}&=0 \Rightarrow \sum_i \left[2\left(z_i-z_c\right)+\sigma_x^{-2}\left(x_i-x_c\right)^2+\sigma_y^{-2}\left(y_i-y_c\right)^2\right]\left(y_i-y_c\right)=0\\
\frac{\partial L}{\partial x_c}&=0 \Rightarrow \sum_i \left[2\left(z_i-z_c\right)+\sigma_x^{-2}\left(x_i-x_c\right)^2+\sigma_y^{-2}\left(y_i-y_c\right)^2\right]=0
\end{aligned}$$

$$r_i=z_i-f_i(\hat{x}_0,\hat{y}_0,\hat{z}_0)=\begin{Bmatrix}\dfrac{\partial f_i}{\partial x_0} & \dfrac{\partial f_i}{\partial y_0} & \dfrac{\partial f_i}{\partial z_0}\end{Bmatrix}\begin{Bmatrix}\delta x_0\\ \delta y_0\\ \delta z_0\end{Bmatrix}=F_i^{\mathrm{T}}\Delta$$

Star Position Estimation Improvements

- Non-Linear 2-D Gaussian Fitting

$$R = \begin{Bmatrix} r_1 \\ \vdots \\ r_n \end{Bmatrix} = \begin{bmatrix} f_{1x} & f_{1y} & f_{1z} \\ \vdots & \vdots & \vdots \\ f_{nx} & f_{ny} & f_{nz} \end{bmatrix} \begin{Bmatrix} \delta x_0 \\ \delta y_0 \\ \delta z_0 \end{Bmatrix} = F\Delta$$

$$\Delta_{k+1} = [F_k^{\mathrm{T}} F_k]^{-1} F_k^{\mathrm{T}} R_k$$

Star Position Estimation Improvements

- Linear Least Squares Log of Gaussian Estimation(LOG-LSQ)

$$f\left(\mathbf{X},\mathbf{X}_c\right)=\frac{1}{2\pi\sqrt{\det R}}e^{-\frac{1}{2}(\mathbf{X}-\mathbf{X}_c)^T R^{-1}(\mathbf{X}-\mathbf{X}_c)}$$

$\mathbf{X}=[x\ y]^T$: pixel location

$\mathbf{X}_c=[x_c\ y_c]^T$: pixel location of the centroid

R : 2×2 symmetric covariance matrix

⇨ $$-\ln f=-\ln\frac{1}{2\pi\sqrt{\det R}}+\frac{1}{2}\left(\mathbf{X}-\mathbf{X}_c\right)^T R^{-1}\left(\mathbf{X}-\mathbf{X}_c\right)$$

$$-\ln f=\underbrace{-\ln\frac{1}{2\pi\sqrt{\det R}}+\frac{1}{2}\mathbf{X}_c^T D\mathbf{X}_c}_{a_0}+\frac{1}{2}\mathbf{X}^T D\mathbf{X}-\mathbf{X}_c^T D\mathbf{X}$$

where $D=R^{-1}$.

$$-\ln f=a_0+\frac{1}{2}\mathbf{X}^T D\mathbf{X}-\mathbf{X}_c^T D\mathbf{X}$$

Star Position Estimation Improvements

- Linear Least Squares Log of Gaussian Estimation(LOG-LSQ)

$$-\ln f = a_0 + \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

where $[a_1\ a_2] = \mathbf{X}_c^T D$

$$-\ln f = a_0 - a_1 x - a_2 y + \frac{1}{2} d_{11} x^2 + \frac{1}{2} d_{12} xy + \frac{1}{2} d_{21} xy + \frac{1}{2} d_{22} y^2$$

$$\begin{bmatrix} -\ln f_1 \\ -\ln f_2 \\ \vdots \\ -\ln f_n \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -y_1 & \frac{1}{2}x_1^2 & \frac{1}{2}x_1 y_1 & \frac{1}{2}x_1 y_1 & \frac{1}{2}y_1^2 \\ 1 & -x_2 & -y_2 & \frac{1}{2}x_2^2 & \frac{1}{2}x_2 y_2 & \frac{1}{2}x_2 y_2 & \frac{1}{2}y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -x_n & -y_n & \frac{1}{2}x_n^2 & \frac{1}{2}x_n y_n & \frac{1}{2}x_n y_n & \frac{1}{2}y_n^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ d_{11} \\ d_{12} \\ d_{21} \\ d_{22} \end{bmatrix}$$

$$\tilde{f} = A' \mathbf{z}'$$

Star Position
Estimation Improvements

- Linear Least Squares Log of Gaussian Estimation(LOG-LSQ)

$$R = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

where ρ*, The correlation between the x and y directions*

$$\begin{bmatrix} -\ln f_1 \\ -\ln f_2 \\ \vdots \\ -\ln f_n \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -y_1 & \frac{1}{2}x_1^2 & x_1y_1 & \frac{1}{2}y_1^2 \\ 1 & -x_2 & -y_2 & \frac{1}{2}x_2^2 & x_2y_2 & \frac{1}{2}y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -x_n & -y_n & \frac{1}{2}x_n^2 & x_ny_n & \frac{1}{2}y_n^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ d_{11} \\ d_2 \\ d_{22} \end{bmatrix}$$

$$\tilde{f} = A\mathrm{z}$$

Star Position
Estimation Improvements

- Linear Least Squares Log of Gaussian Estimation(LOG-LSQ)

$$\begin{bmatrix} -\ln f_1 \\ -\ln f_2 \\ \vdots \\ -\ln f_n \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -y_1 & \frac{1}{2}x_1^2 & x_1 y_1 & \frac{1}{2}y_1^2 \\ 1 & -x_2 & -y_2 & \frac{1}{2}x_2^2 & x_2 y_2 & \frac{1}{2}y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -x_n & -y_n & \frac{1}{2}x_n^2 & x_n y_n & \frac{1}{2}y_n^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ d_{11} \\ d_2 \\ d_{22} \end{bmatrix}$$

$$\tilde{f} = A\mathbf{z}$$

$$\mathbf{z} = \left(A^T A\right)^{-1} A^T F$$

$$X_c^{\ T} D = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{X}_c = \left(\left[a_1 \ a_2\right] D^{-1}\right)^T$$

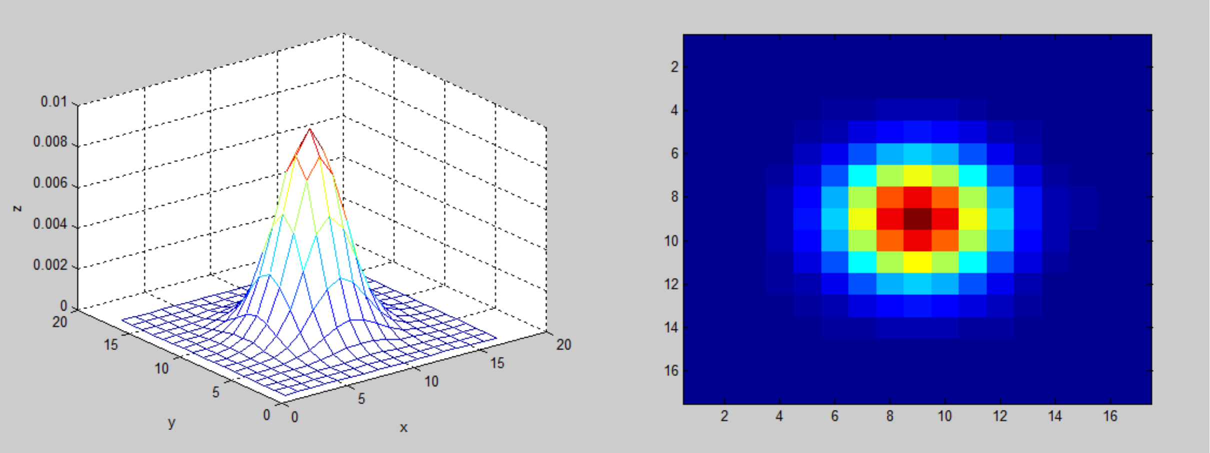
Simulation

- Variable
  - Sigma
  - Magnitude
  - Noise
  - Mask size
  - Weight
  - threshold

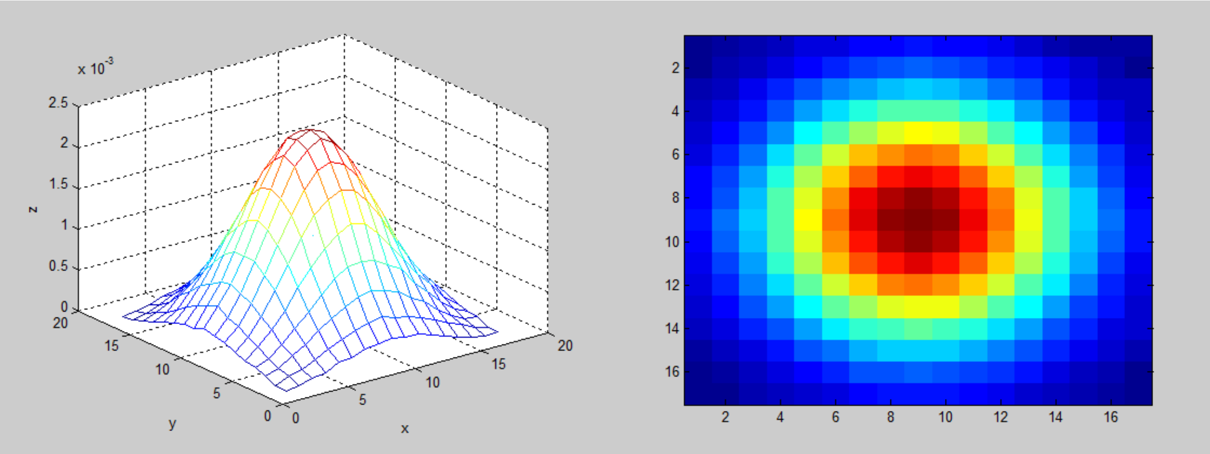
Simulation

- Sigma

Sigma 2

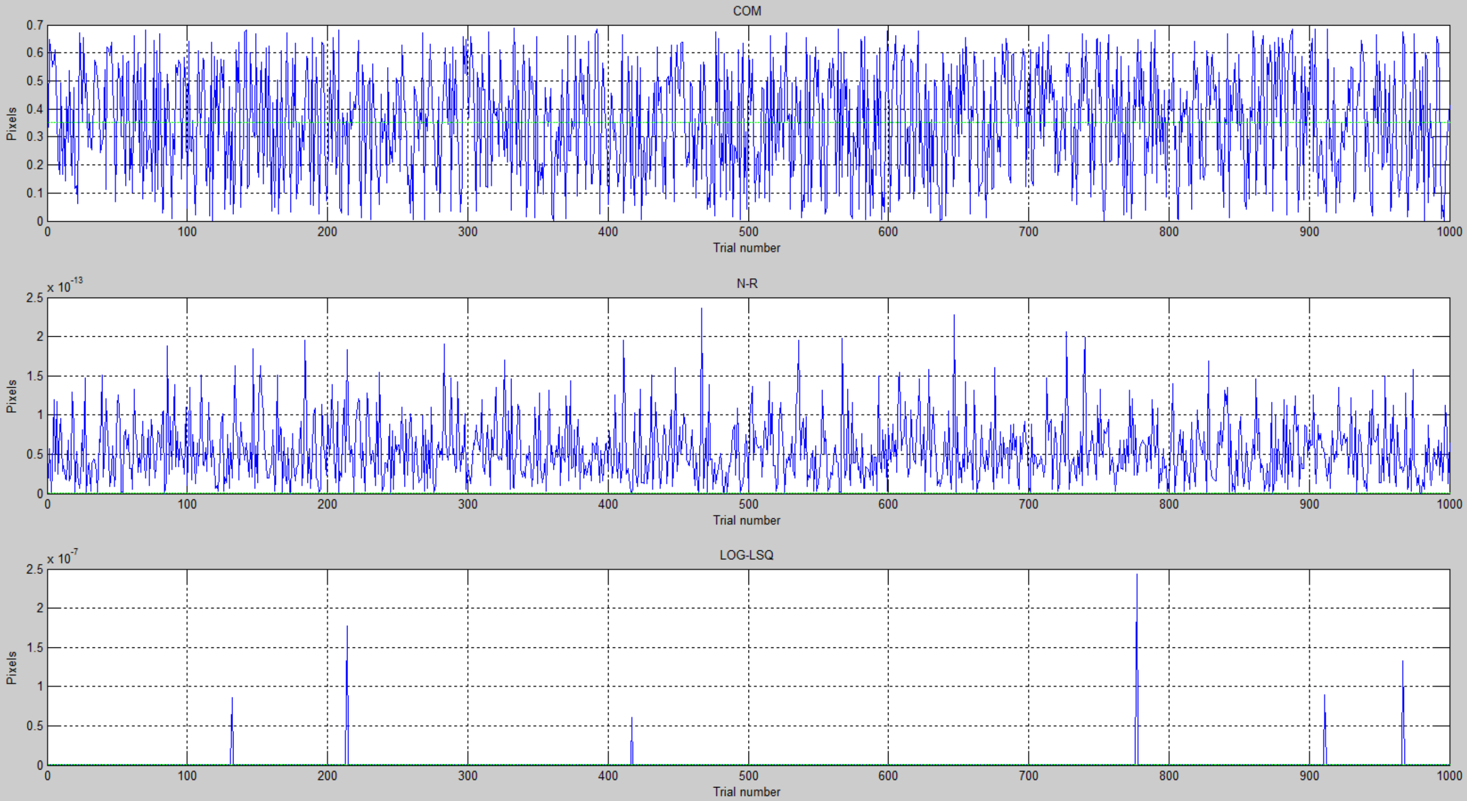


Sigma 4



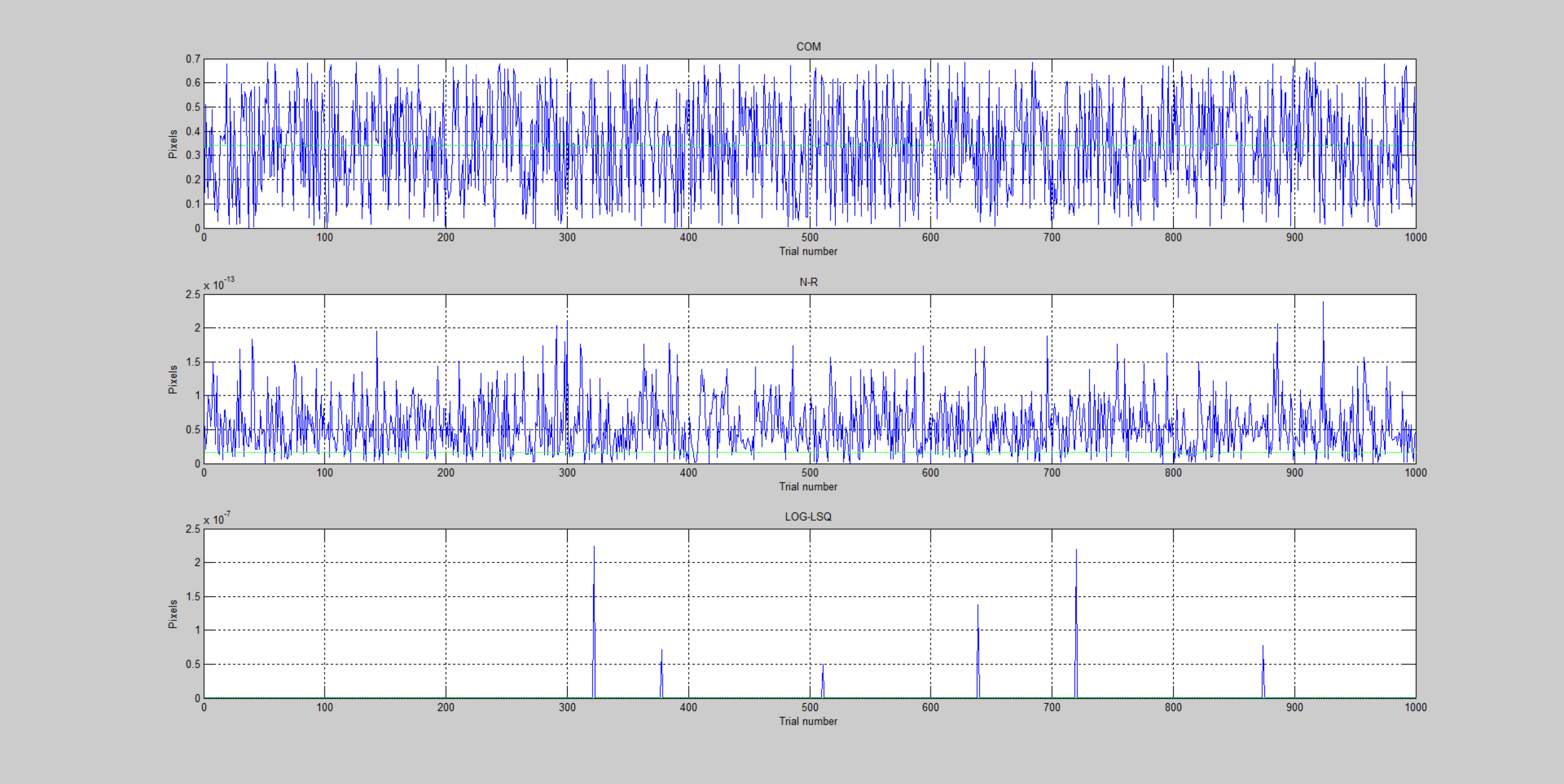
Simulation

- Magnitude (Noise 0%, m -1, Mask 7)



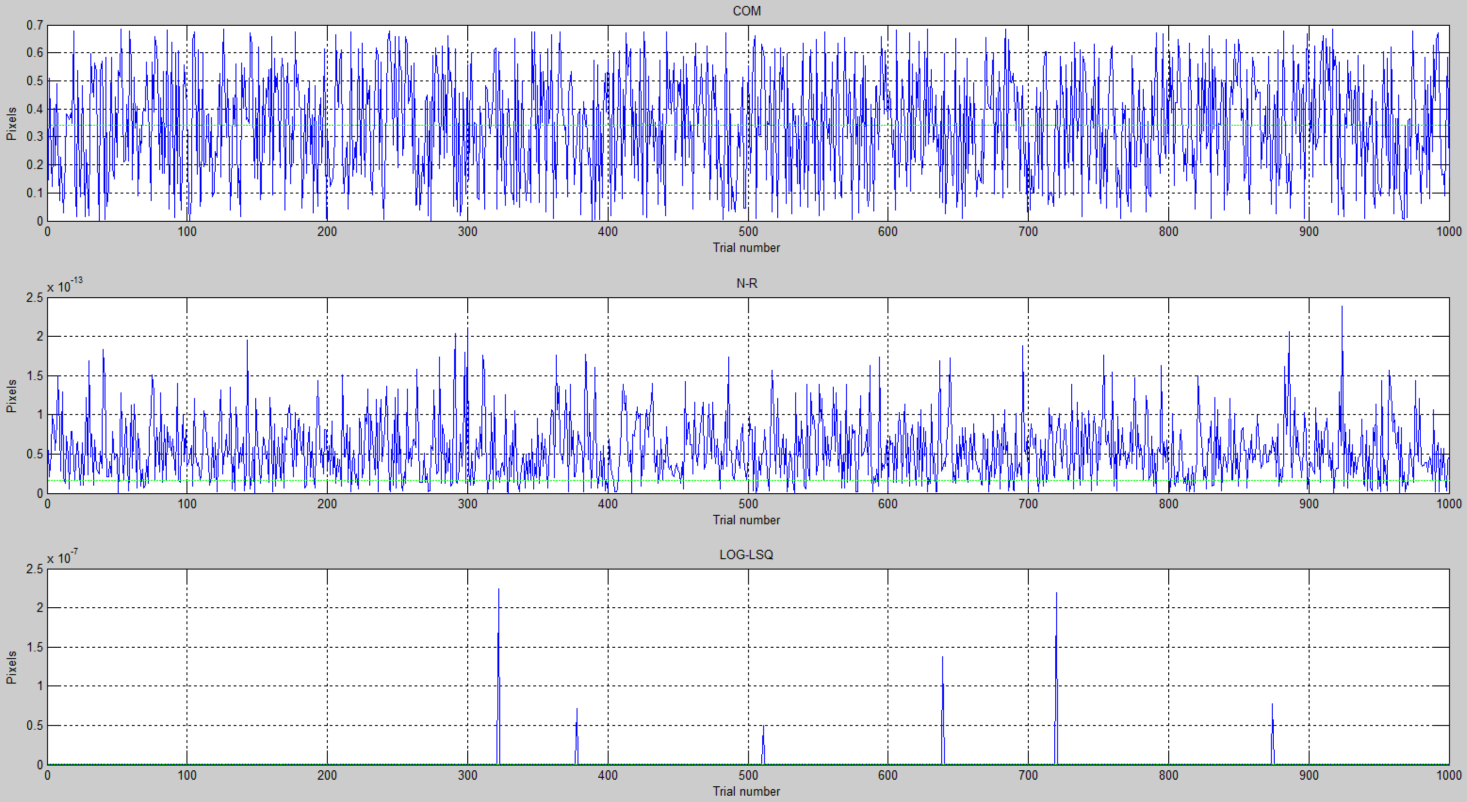
Simulation

- Magnitude (Noise 0%, m 3, Mask 7)



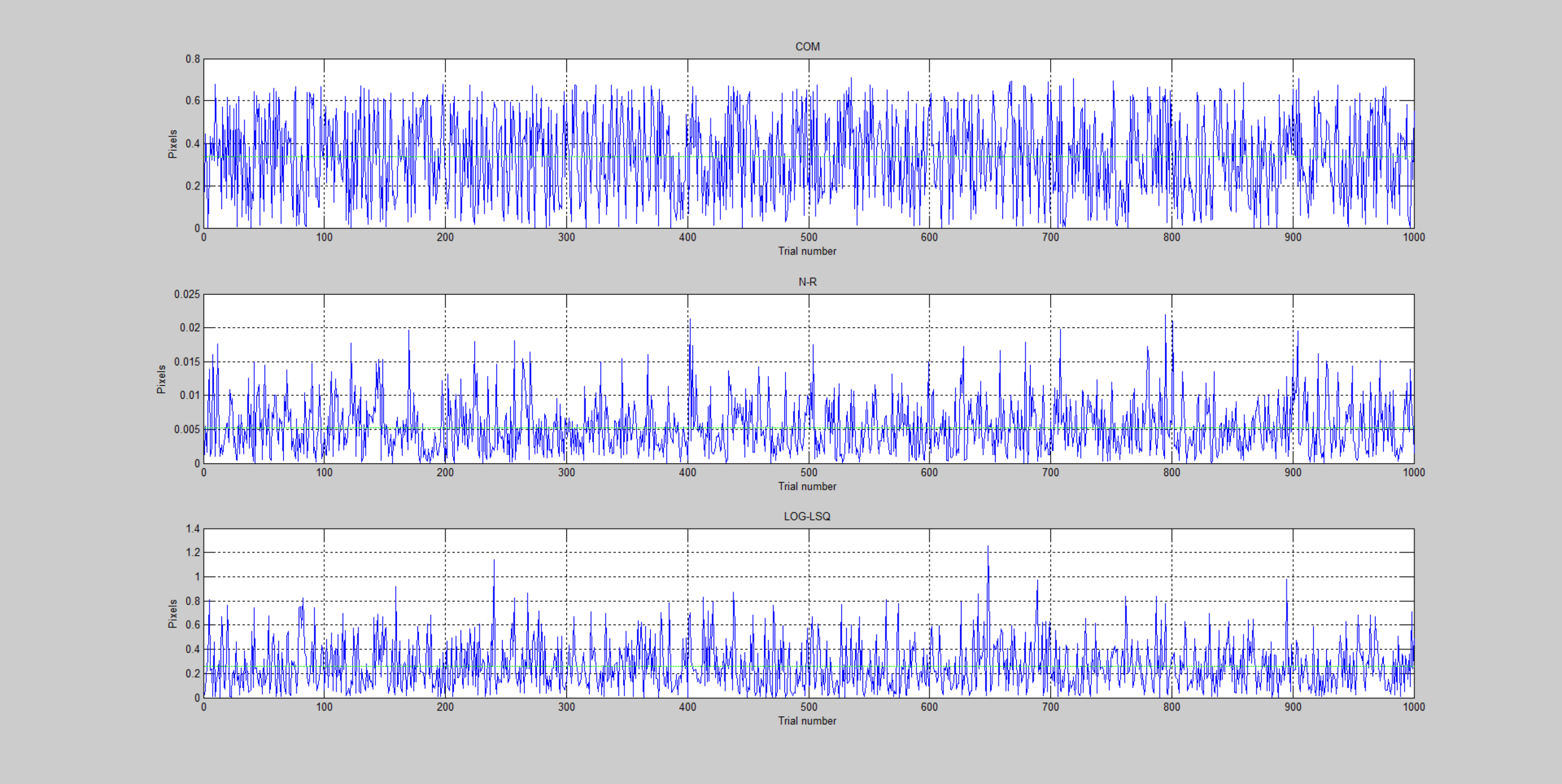
Simulation

- Noise (Noise 0%, m 3, Mask 7)



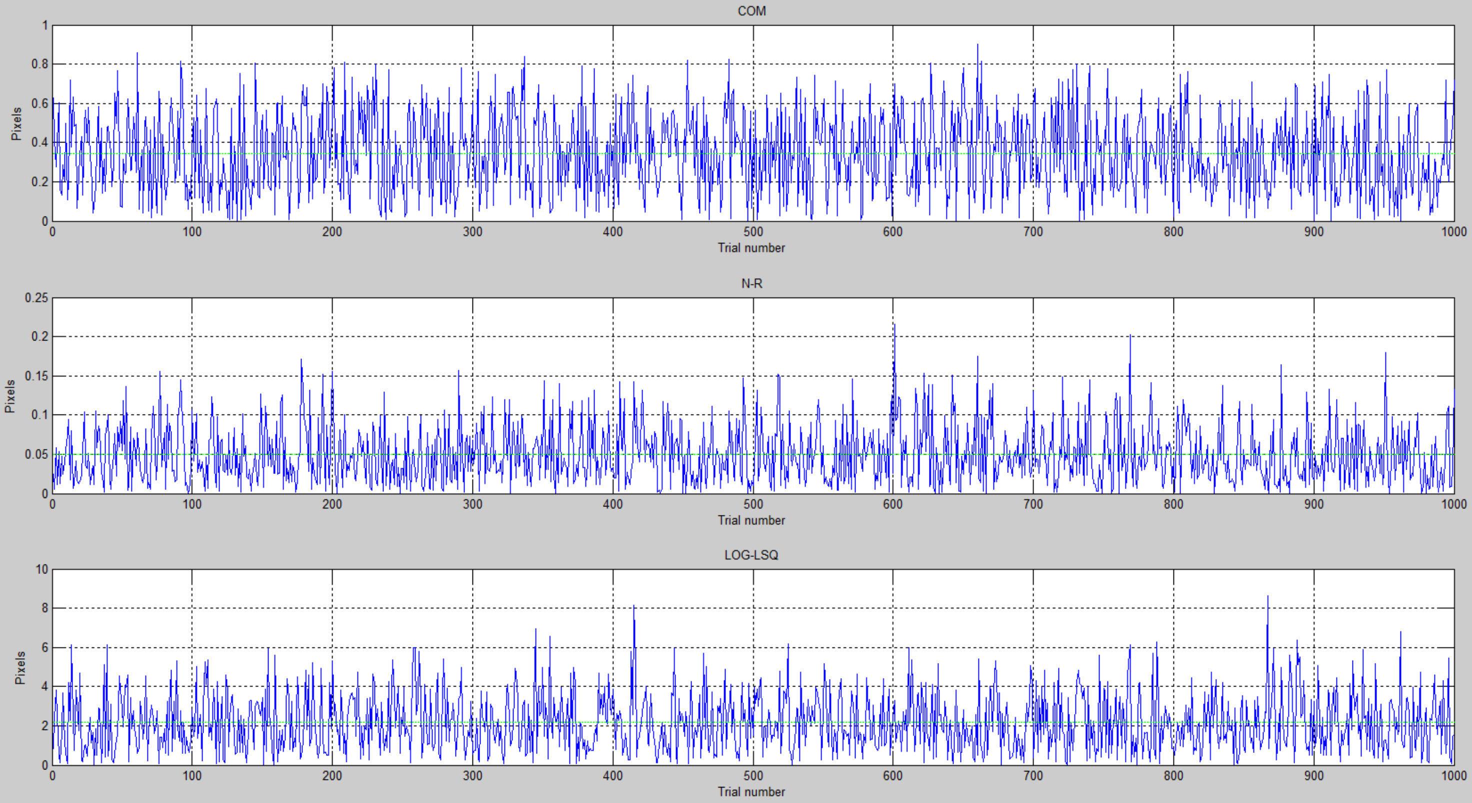
Simulation

- Noise (Noise 1%, m 3, Mask 7)



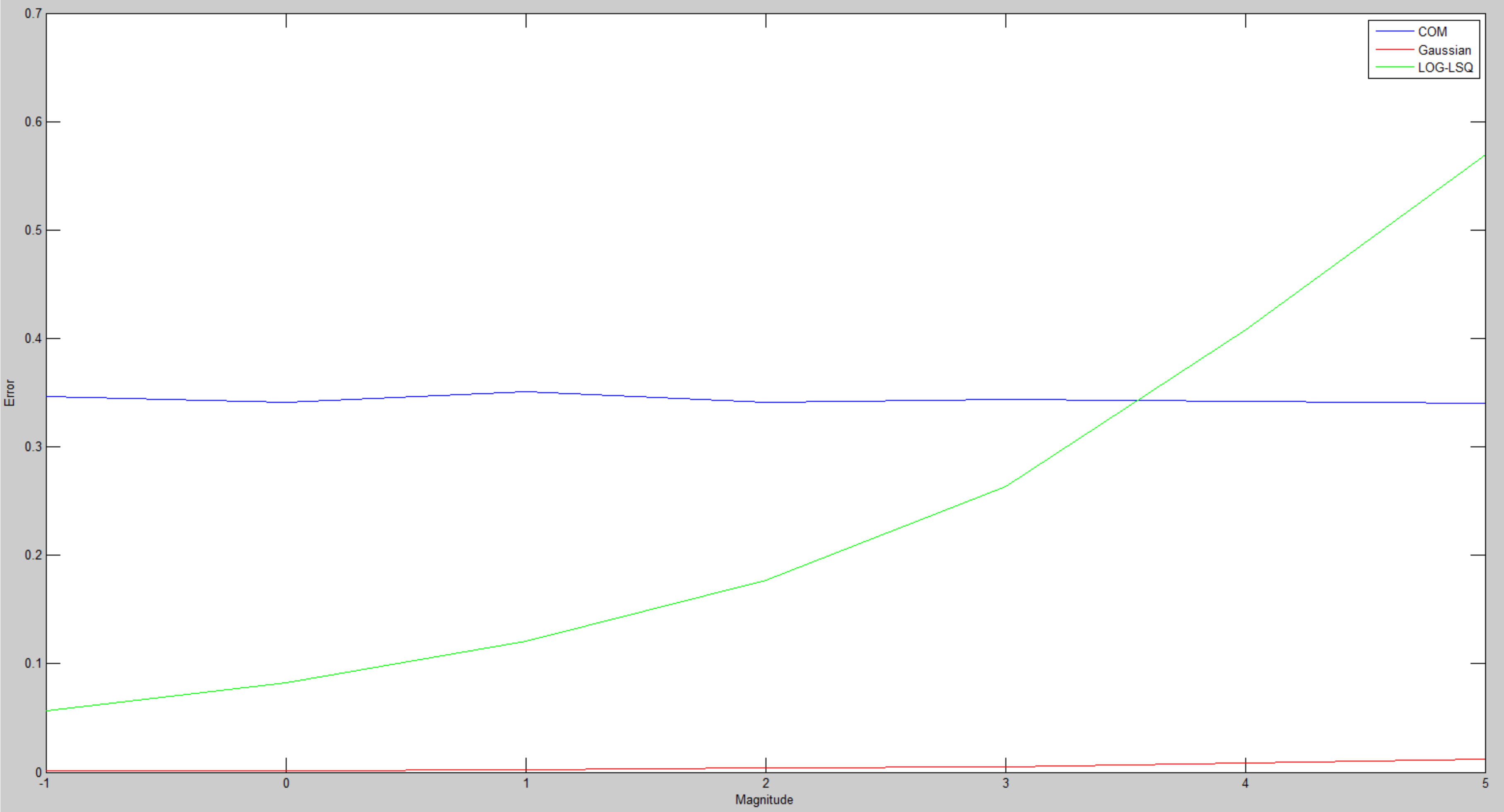
Simulation

- Noise (Noise 10%, m 3, Mask 7)



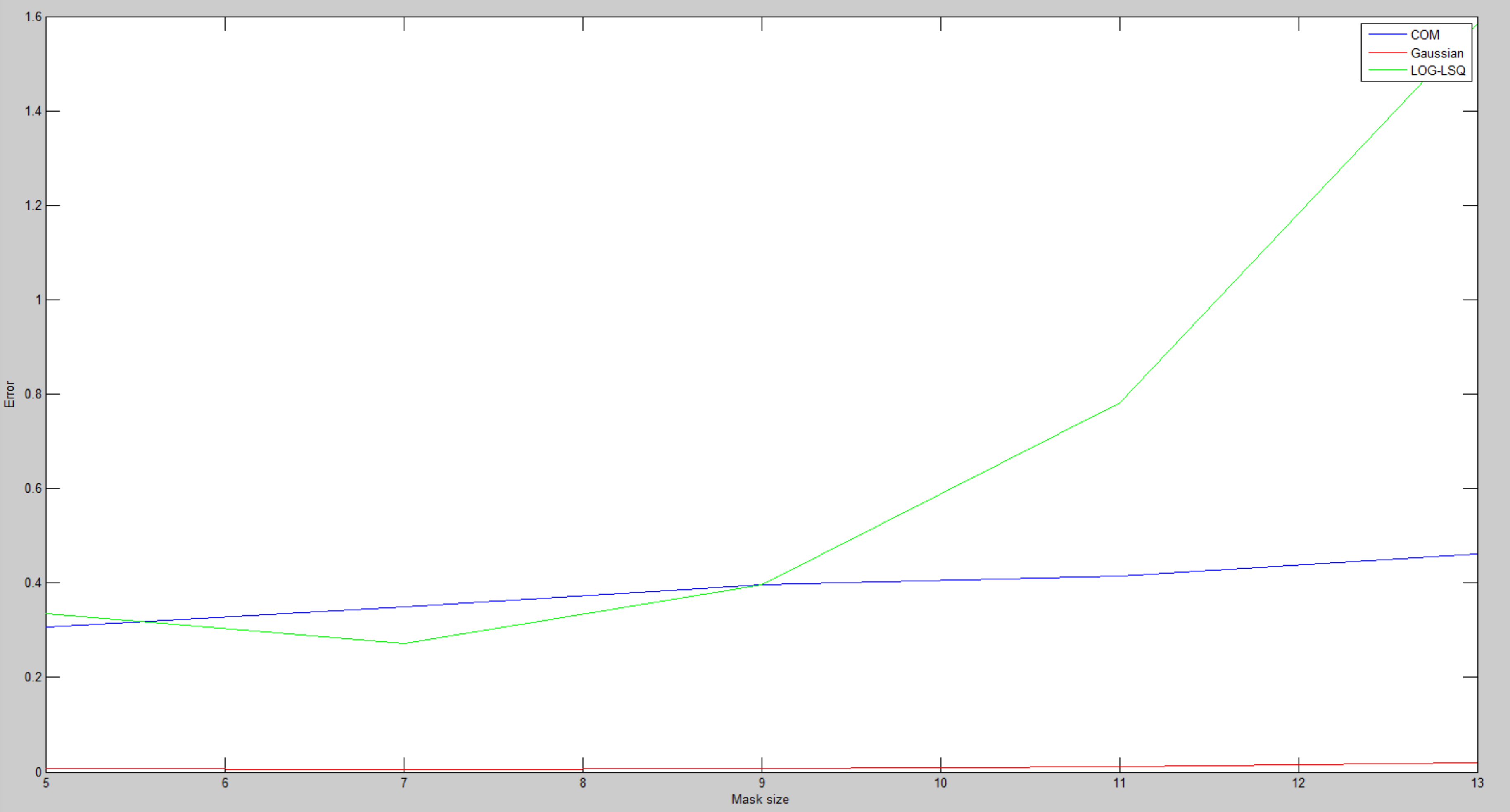
Simulation

- Magnitude Error (Noise 1%, Mask 7)



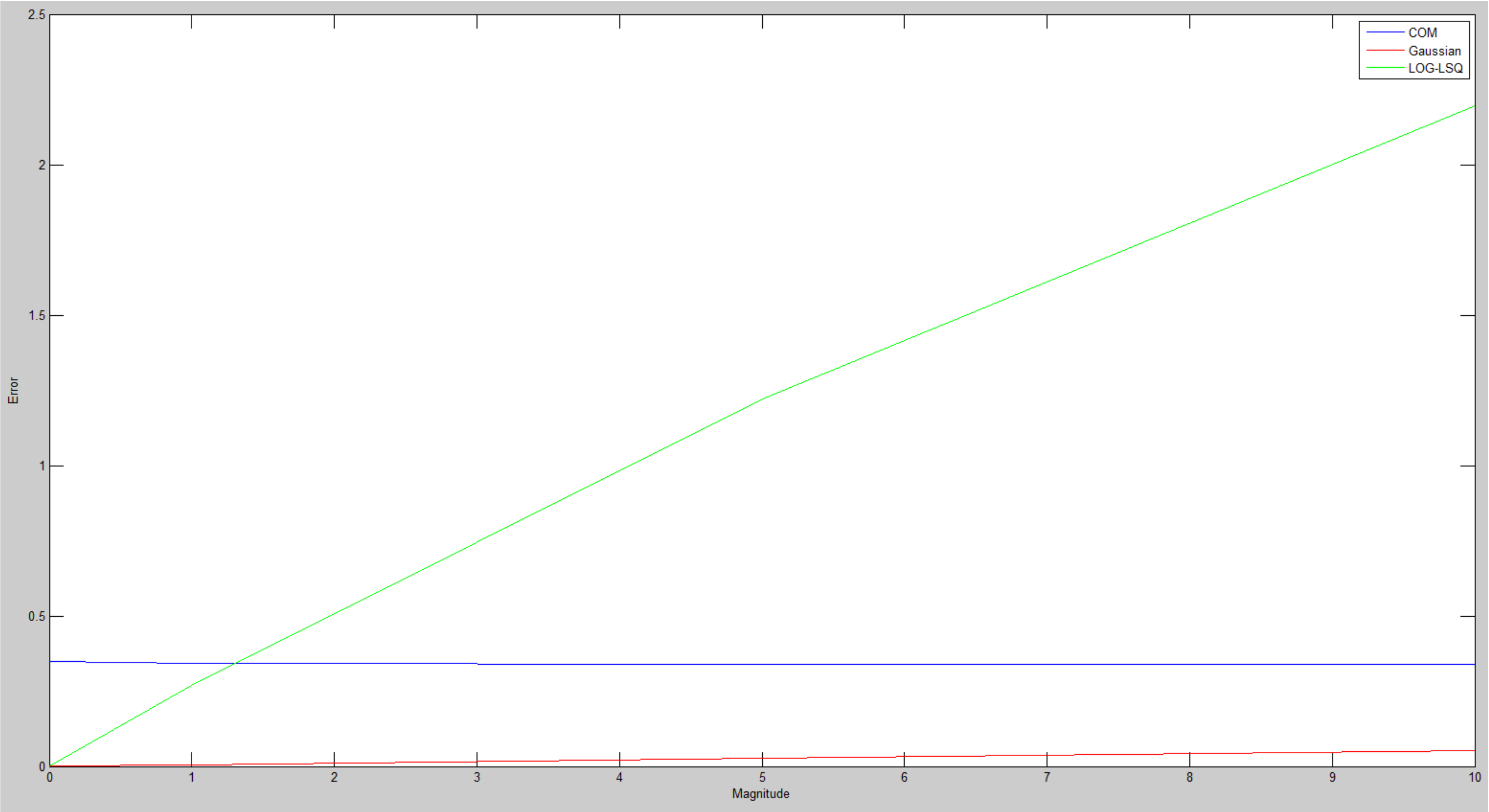
Simulation

- Mask Error (Noise 1%, m 3)



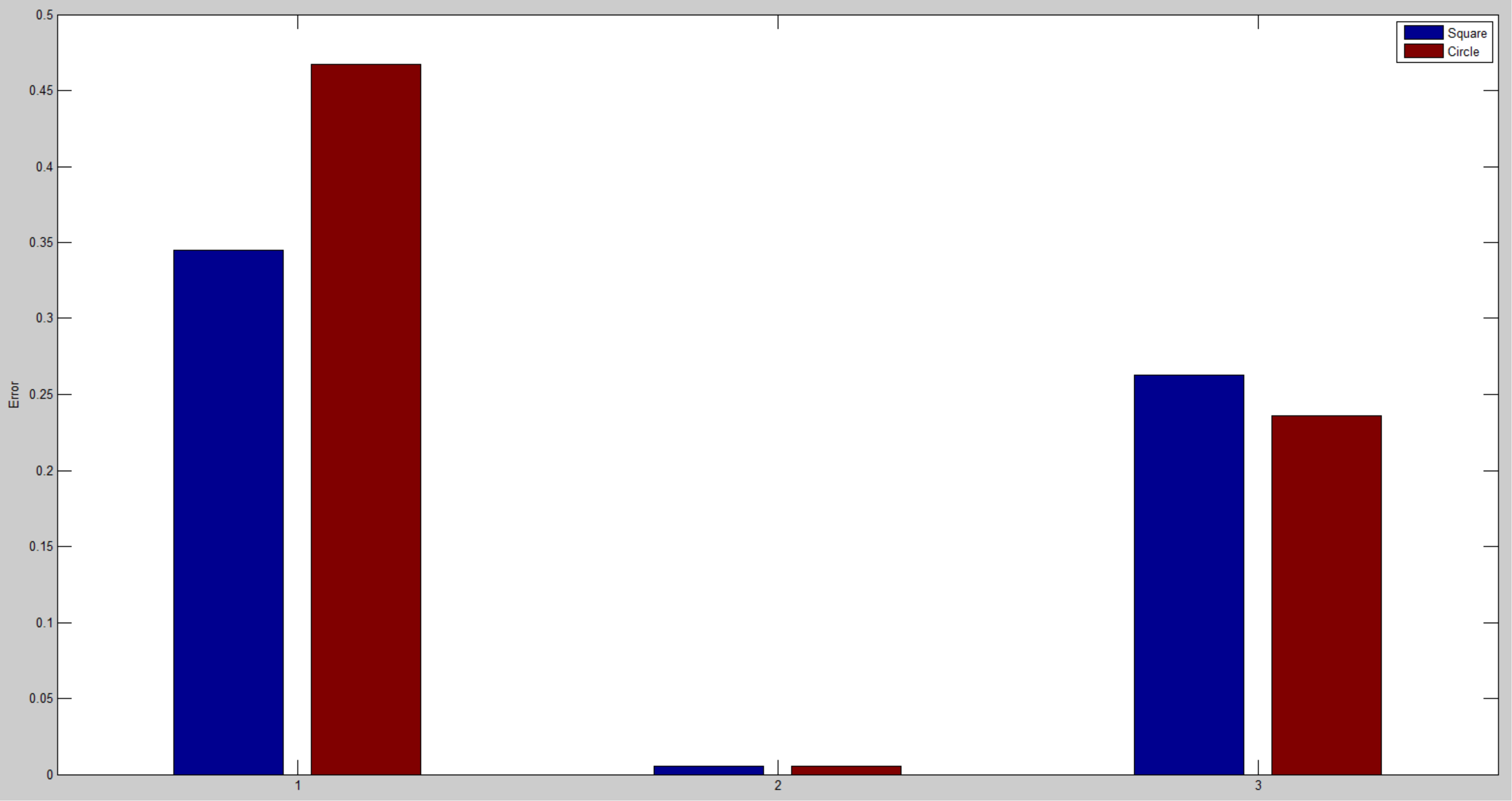
Simulation

- Noise Error (m 3, Mask 7)



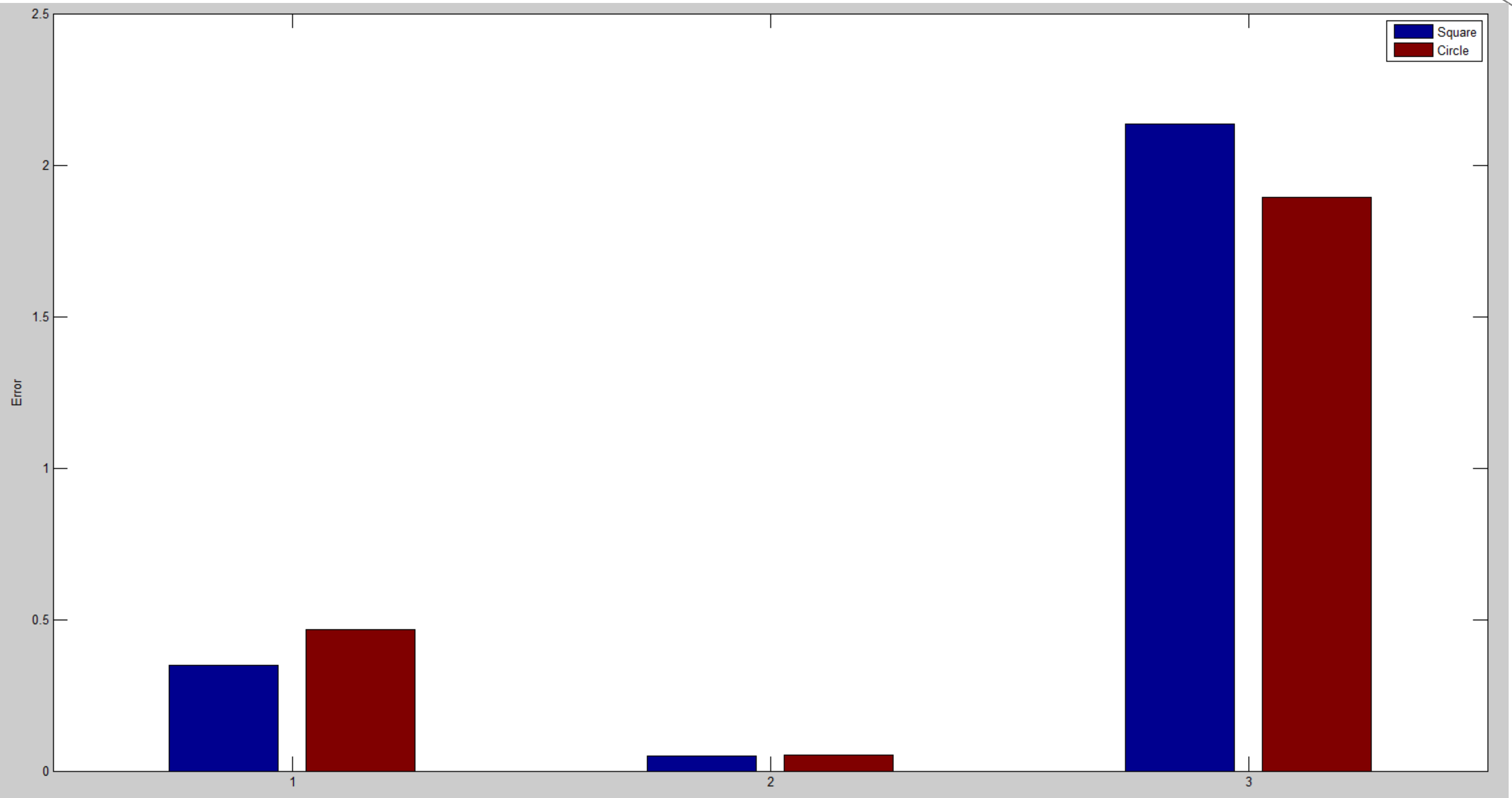
Simulation

- Mask shape (Noise 1%, m 3, Mask 7)



Simulation

- Mask shape (Noise 10%, m 3, Mask 7)



CONCOLUSION

- 수치해석 Method가 가장 정확
- COM은 어느 상황에서도 준수한 성능을 보유
- 방법론마다 Mask shape의 영향이 다름
- Mask size는 유동적으로 계산하는 것이 용이
- Sigma의 값에 따른 마스크 크기 선정