An FFT-Based Technique

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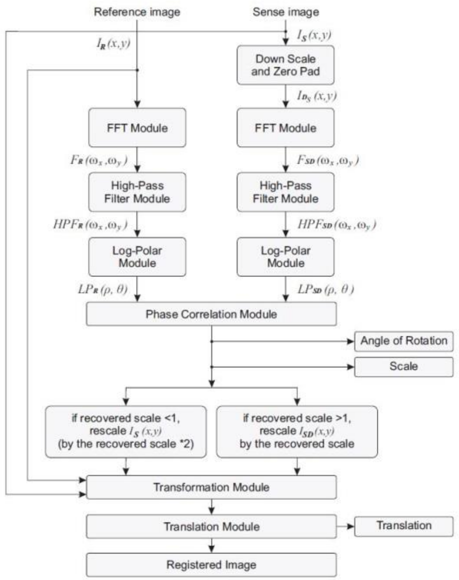
Content

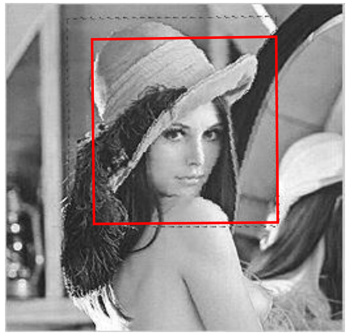
1. What is FFT?

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k\frac{n}{N}} \quad k = 0, \ldots, N-1$$

2. FFT Based Technique



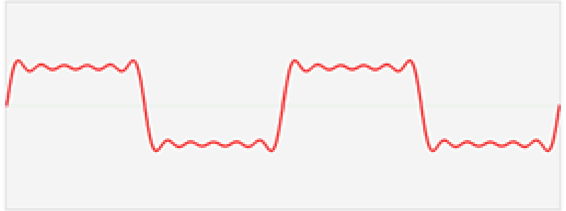
3. Simulation



1. What is FFT?

- Fourier Transform

Based condition : All periodic signal may be expressed sum of sine and cosine



Discrete time Fourier Transform(DFT) is Sampling Data's Fourier Transform.

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k\frac{n}{N}} \quad k = 0, \ldots, N-1$$

at, $e^{-2\pi i(\frac{n}{N})} = \cos(-2\pi i\left(\frac{n}{N}\right)) + \mathrm{i} * \sin(-2\pi i\left(\frac{n}{N}\right))$

DFT's computing time is $O(N^2)$. But FFT's compute time is $O(N\log_2 N)$.

- Removing the portion of the periodic

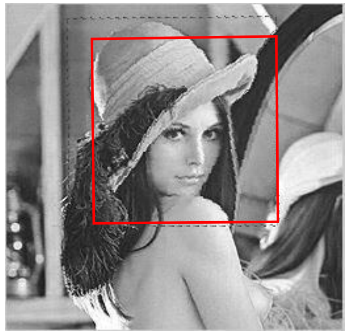
$logn$	n	$nlogn$	n^2	n^3	2^n
0	1	0	1	1	2
1	2	2	4	8	4
2	4	8	16	64	16
3	8	24	64	512	256
4	16	64	256	4,096	65,536
5	32	160	1,024	32,768	4,294,967,296

2. FFT Based Technique

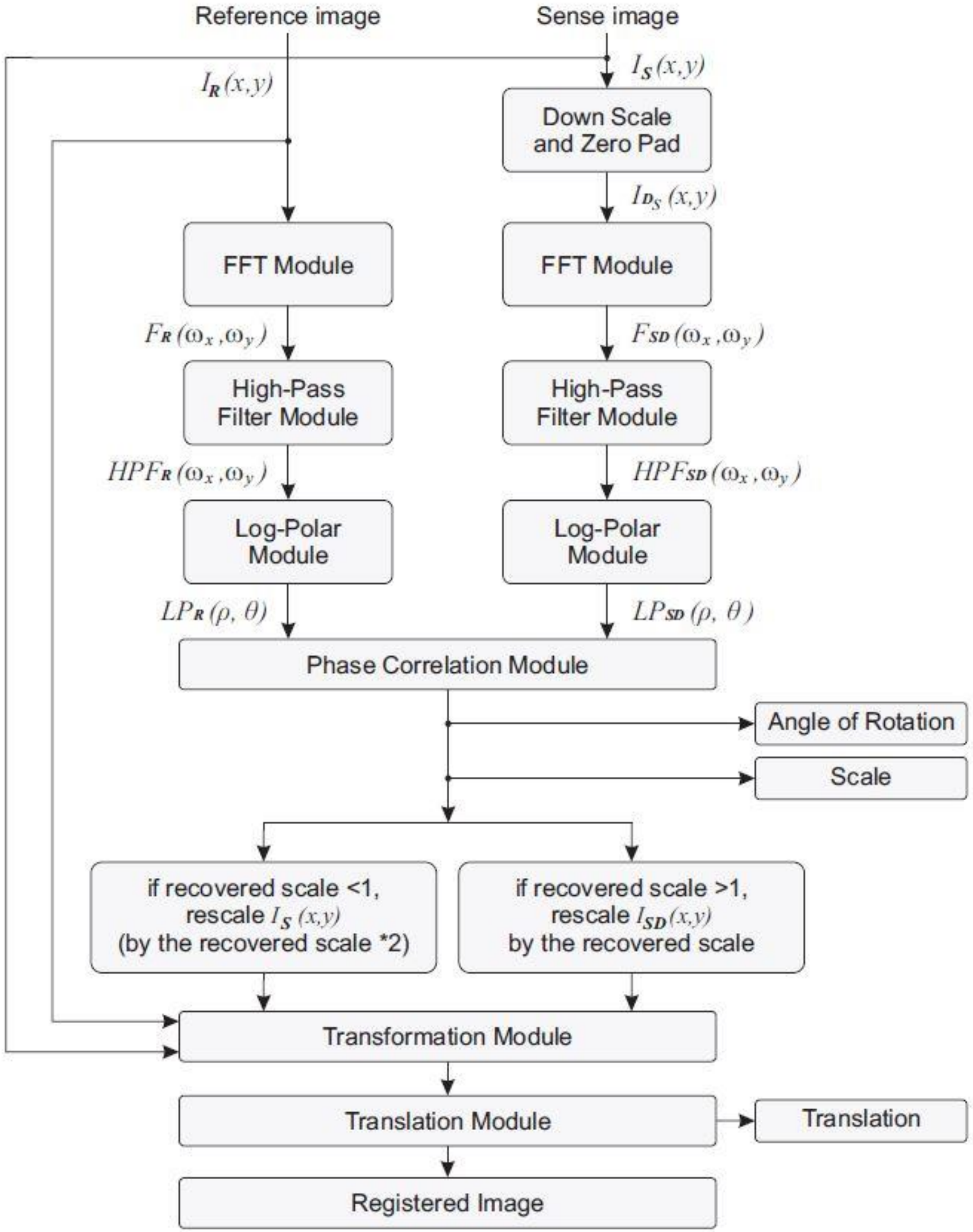
2.1 Block Diagram

Reference image

Sense image



Final image

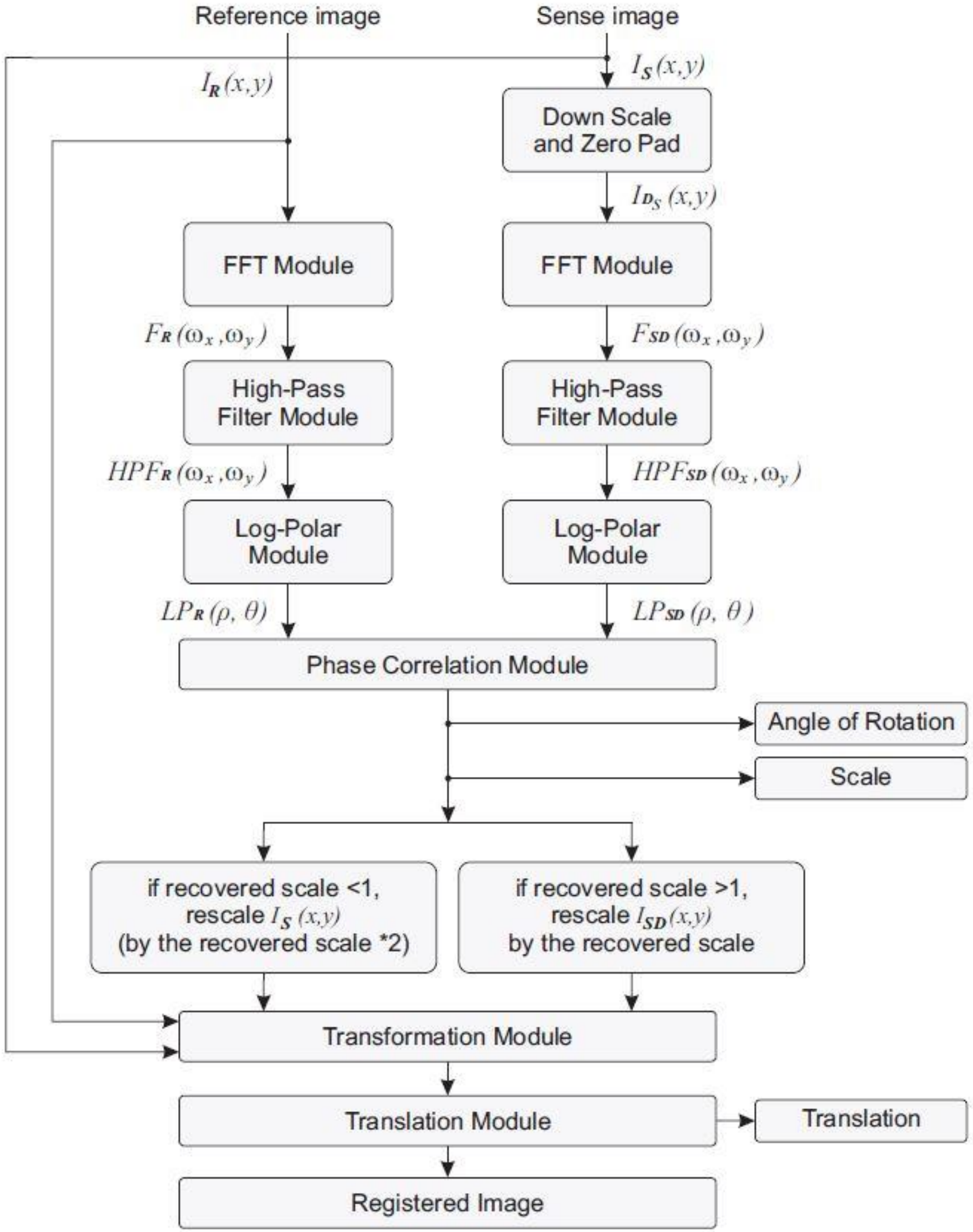


3. Simulation

- Block Diagram

Reference image

Sense image



3. Simulation

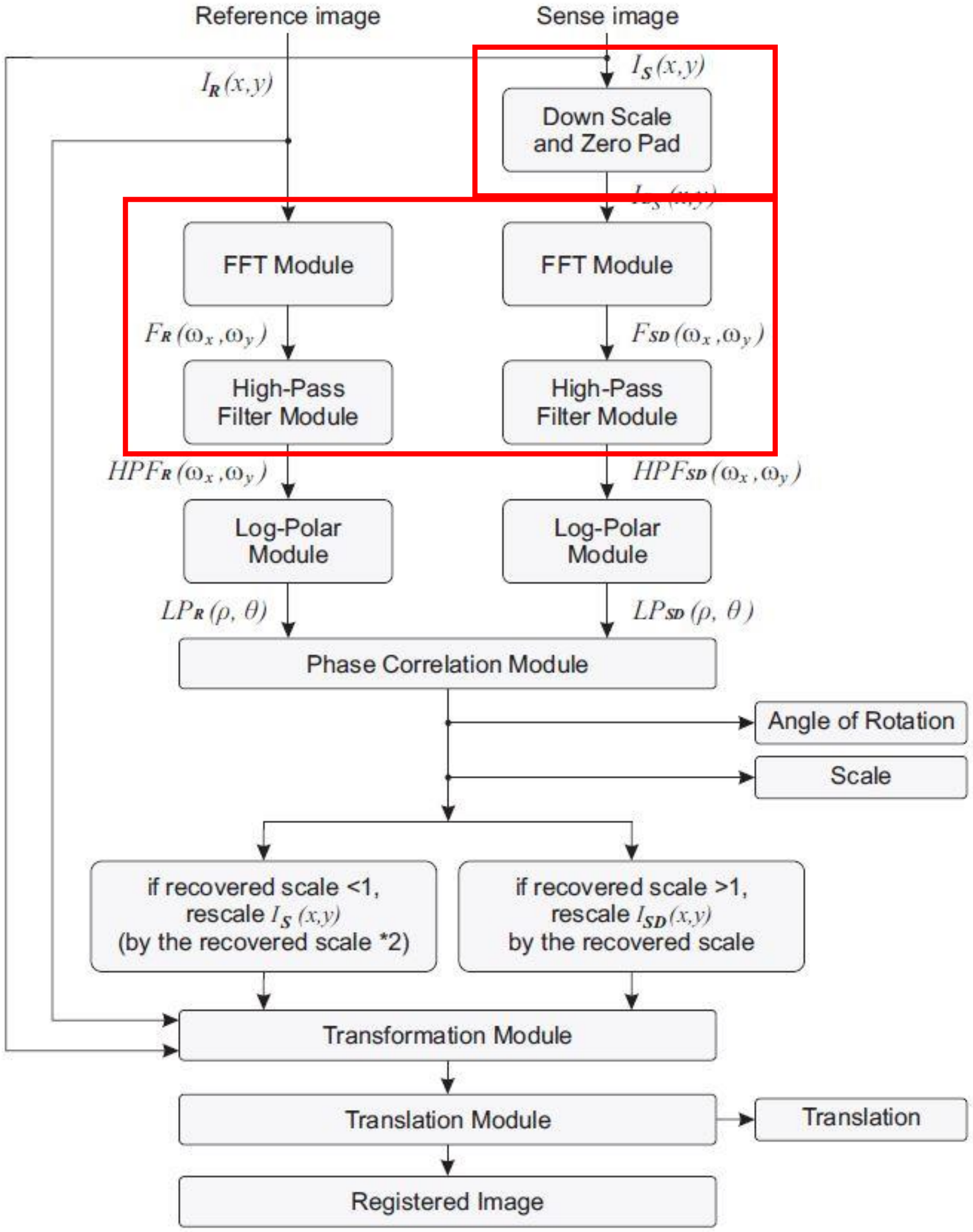
3.1 Zero Pad and FFT

Zero Pad

Cartesian domain | Frequency domain

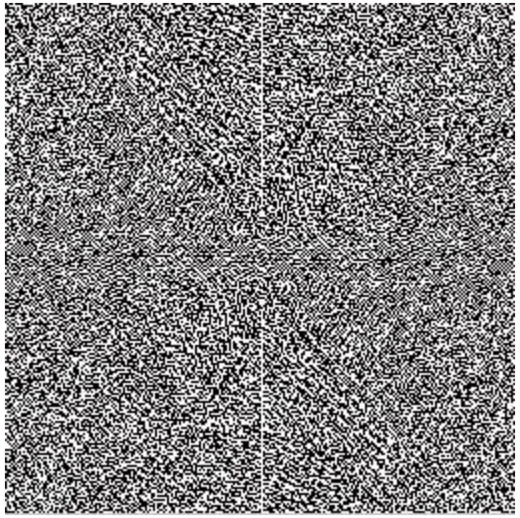
$$f_1(x,y) \Rightarrow F_1(x,y)$$

$$f_2(x,y) \qquad F_2(x,y)$$

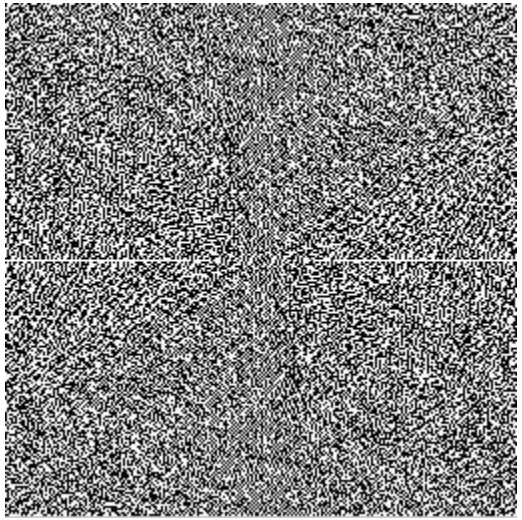


3. Simulation

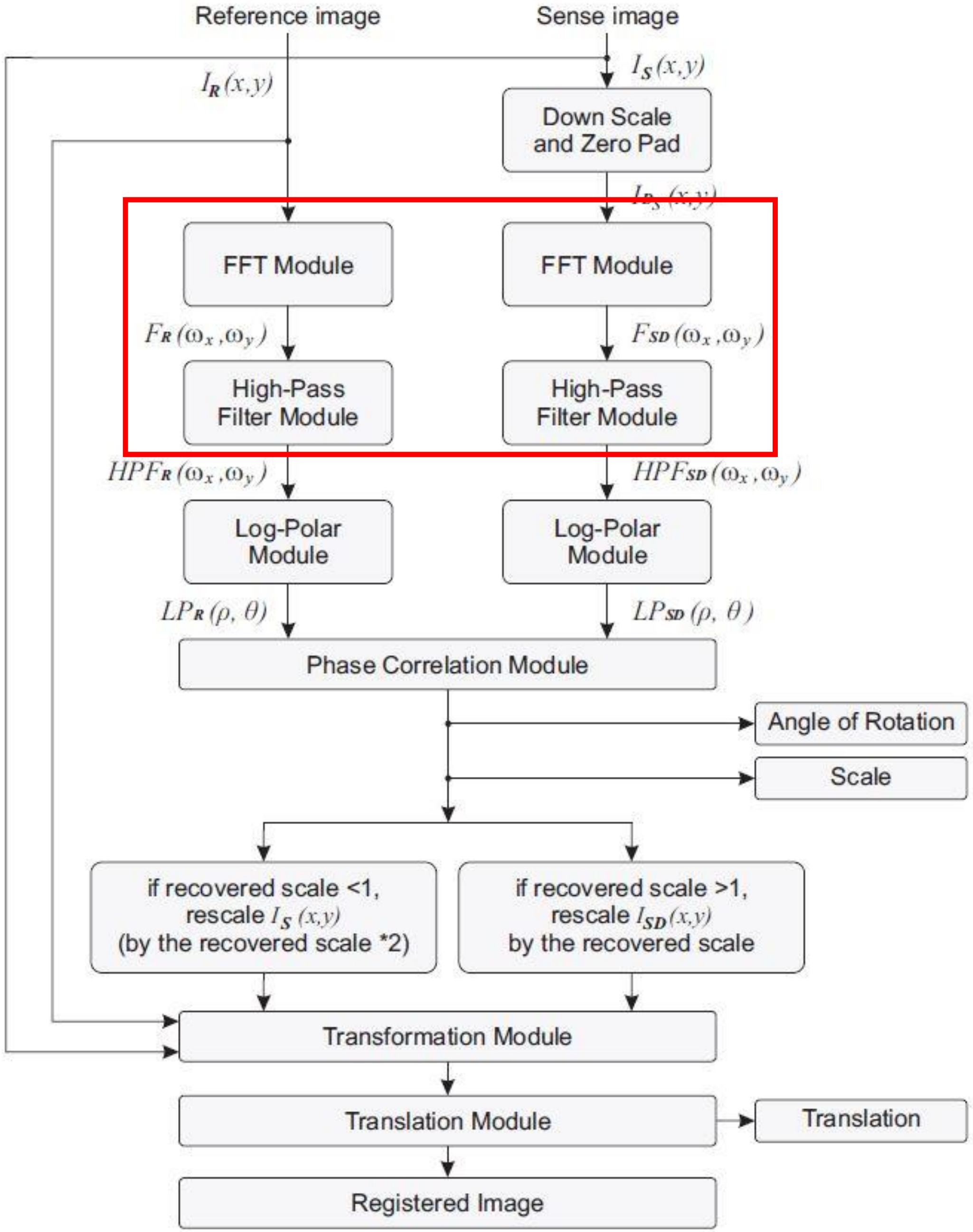
3.1 FFT and High-Pass Filter



F1's FFT and shift



F2's FFT and shift

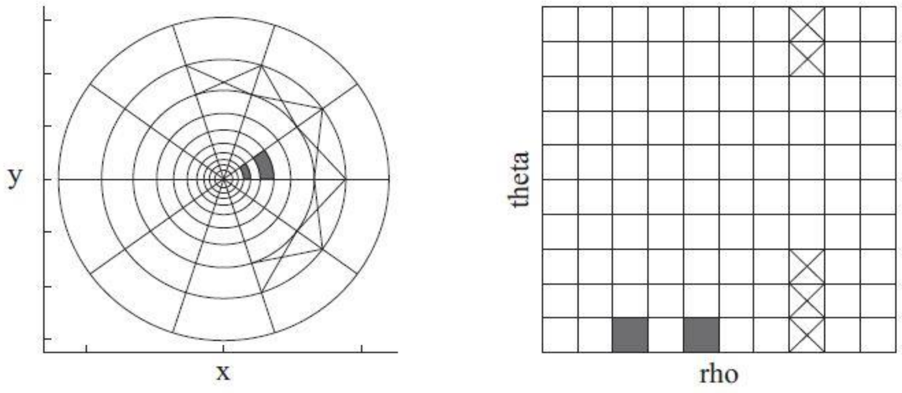


3. Simulation

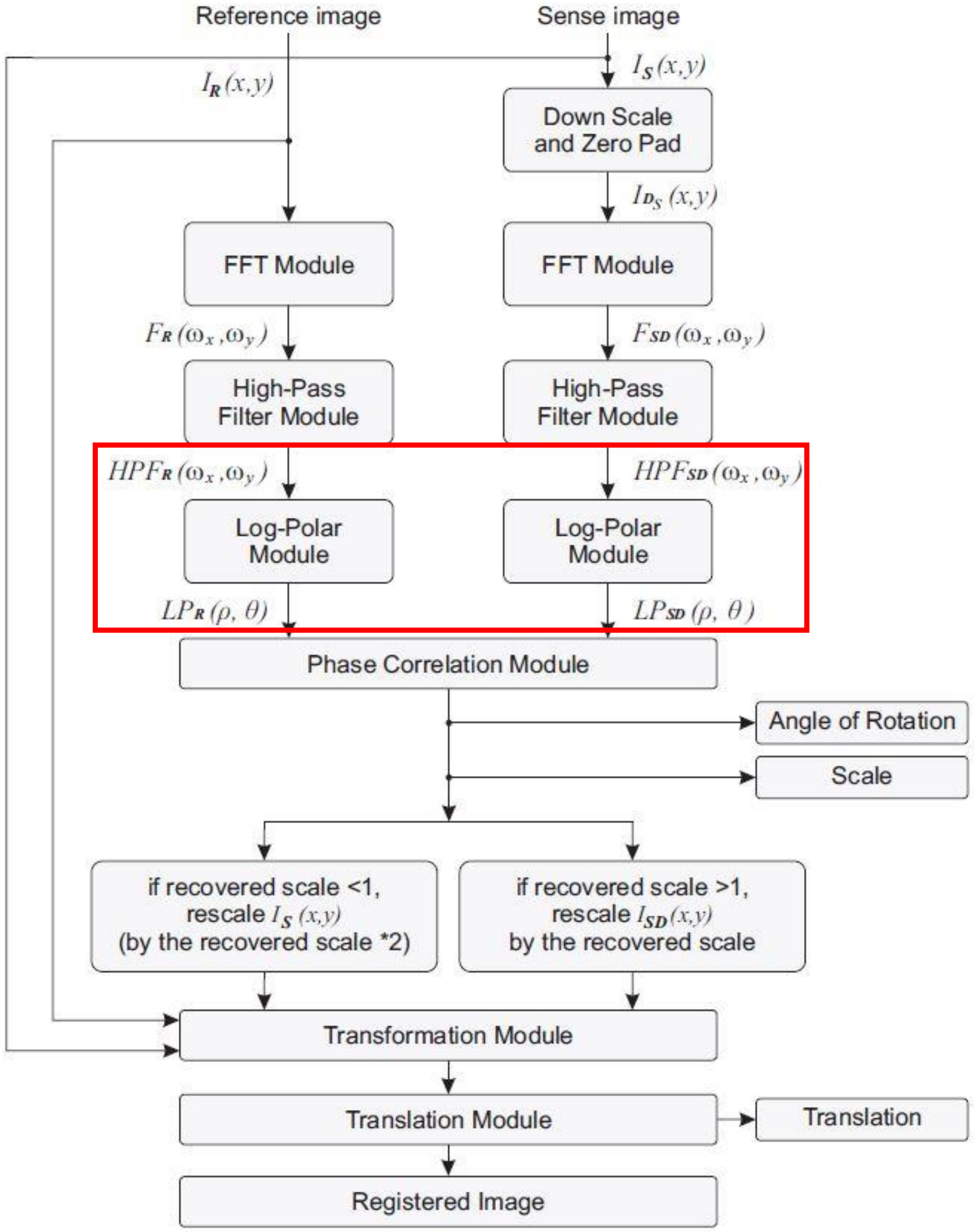
3.2 Log-Polar Transform

What tis the Log Polar Transform

Scaling and rotation in Cartesian domain corresponds to pure translation in log polar domain



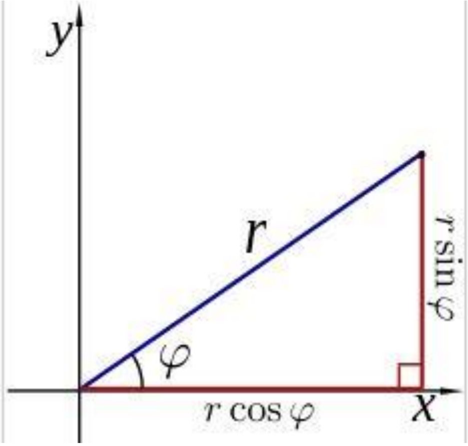
$$(x, y) \rightarrow (\rho, \theta)$$



3. Simulation

3.2 Log-Polar Transform

What tis the Log Polar Transform



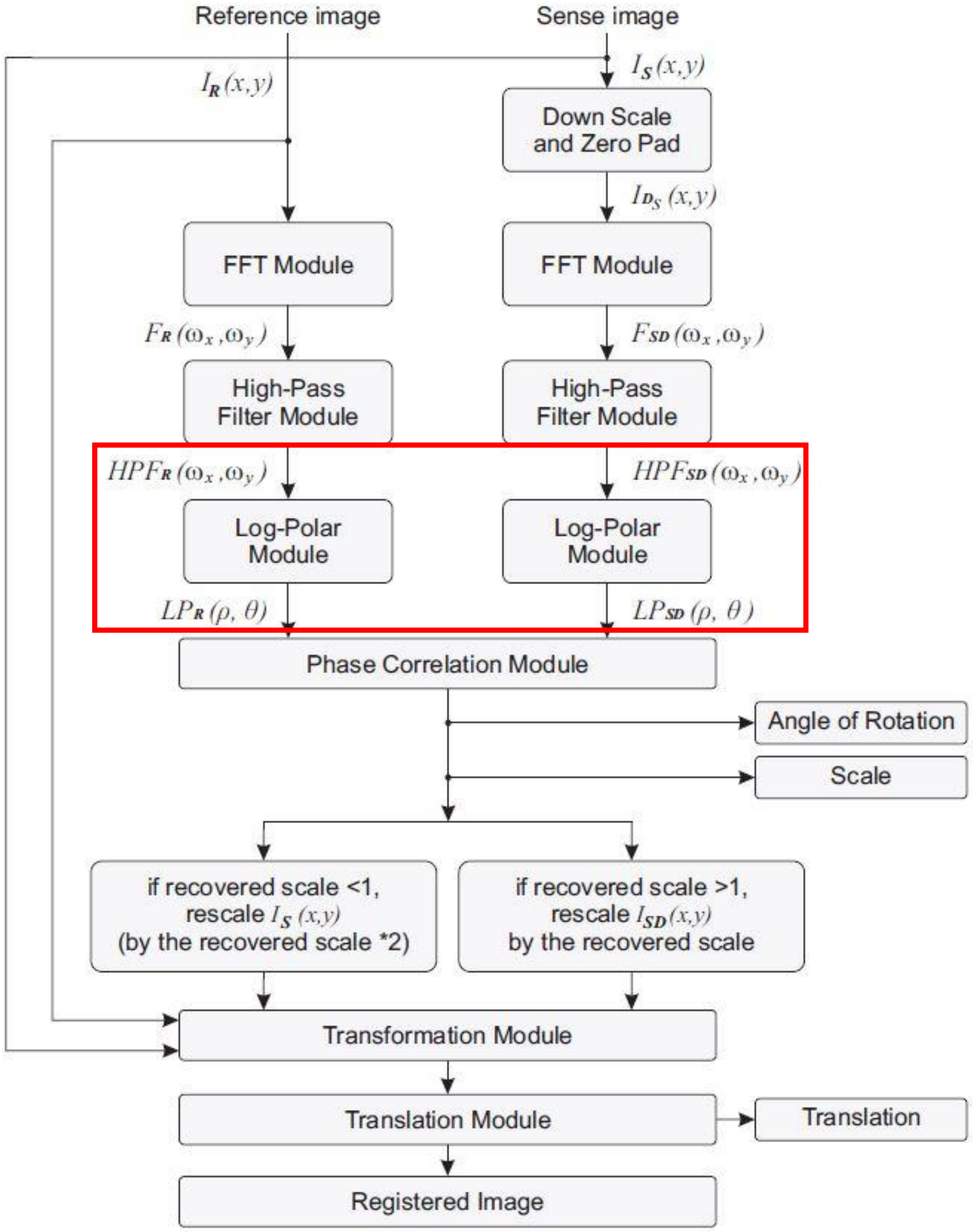
$$(x, y) \rightarrow (\rho, \theta)$$
$$= \left(\sqrt{(x - x_c)^2 + (y - y_c)^2}, \arctan(\frac{y - y_c}{x - x_c})\right)$$

Scaled by a factor of α

$$(\alpha x, \alpha y) \rightarrow (log\alpha + logx, log\alpha + logy)$$

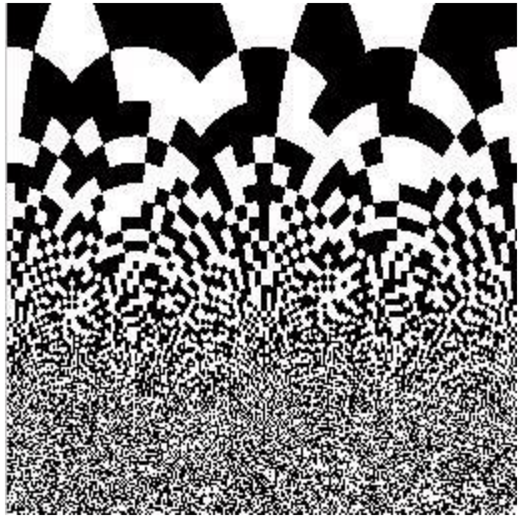
$$\rho' = log\sqrt{(\exp(\rho)\, cos\theta - \Delta x)^2 + (\exp(\rho)\, sin\theta - \Delta y)^2}$$

$$\theta' = arctan\left(\frac{\exp(\rho)\, sin\theta - \Delta y}{\exp(\rho)\, cos\theta - \Delta x}\right) \qquad * \, at \;\; r = e^{\rho}$$

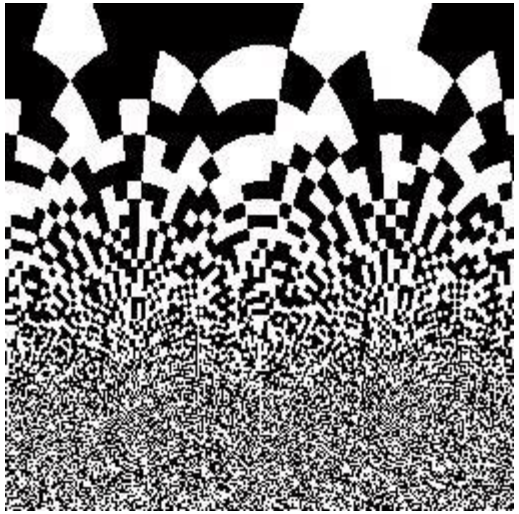


3. Simulation

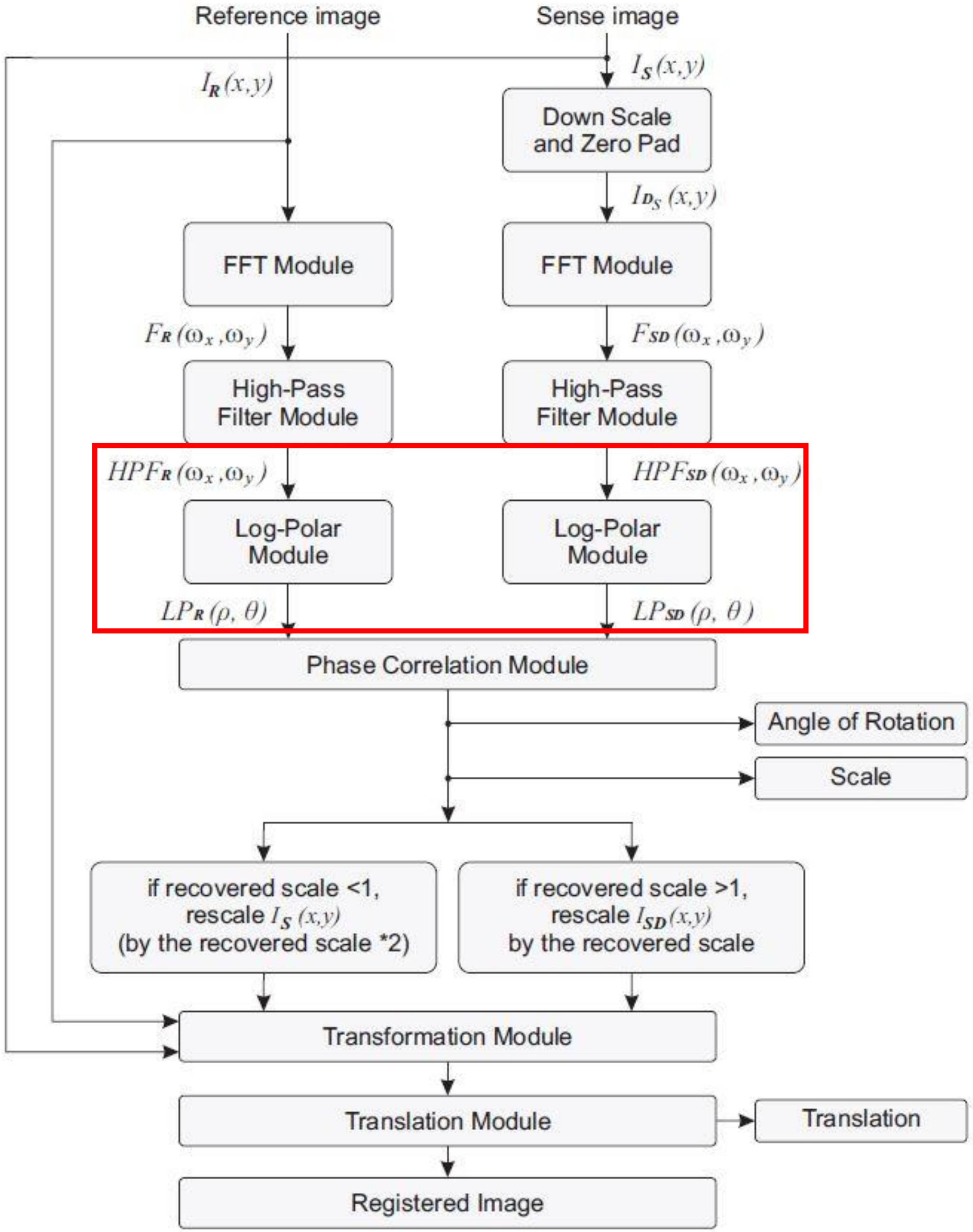
3.2 Log-Polar Transform



L1's FFT and shift



L2's FFT and shift



3. Simulation

3.3 Phase Correlation

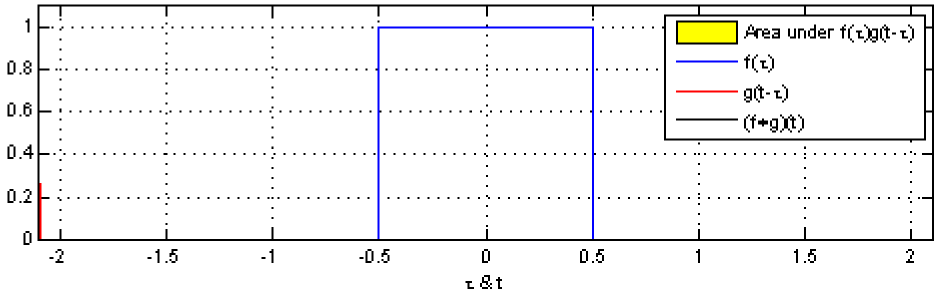
- What is Correlation?

$$(f \star g)[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} f^*[m]\ g[m+n].$$

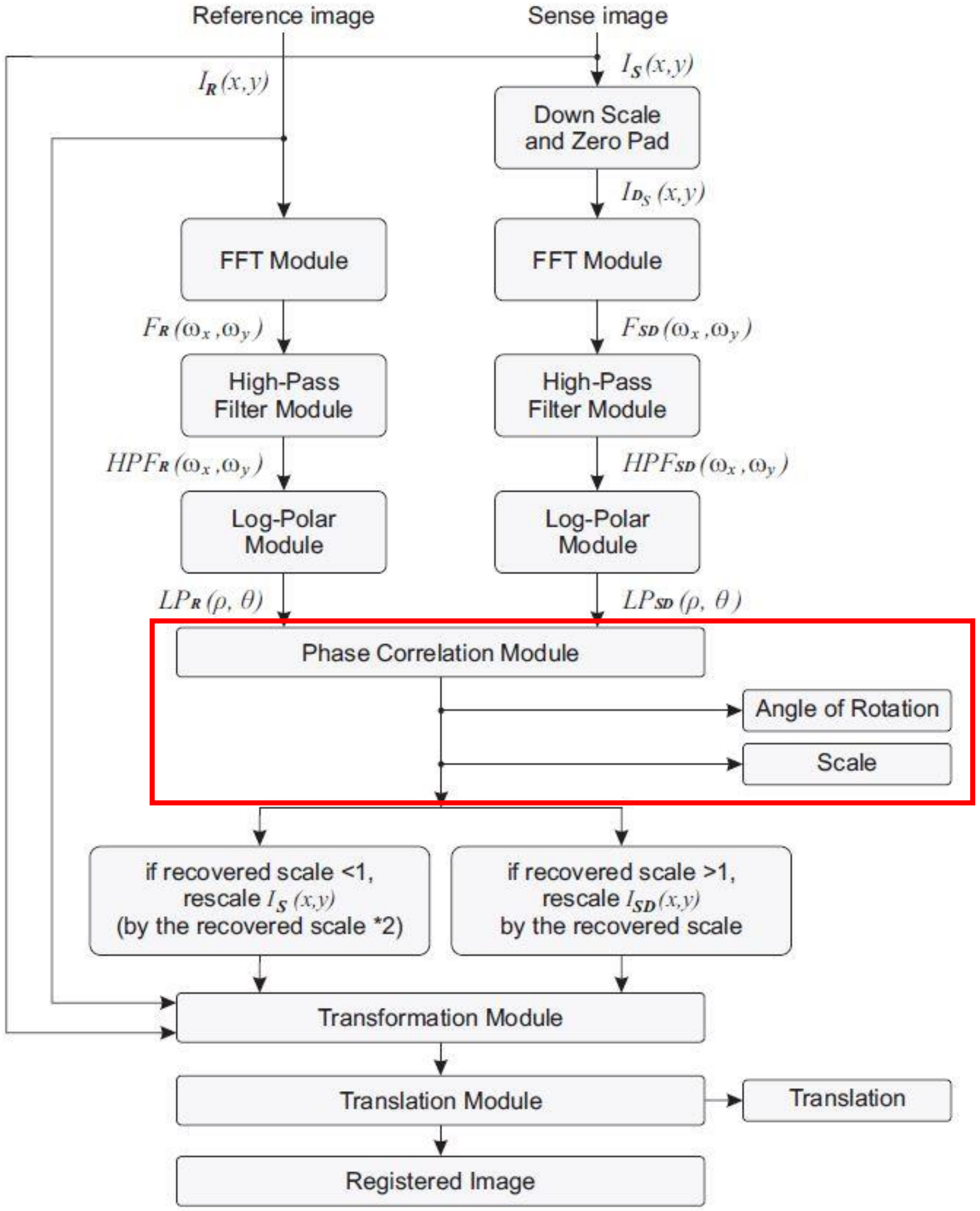
Cross correlation

$$\mathbf{A} \cdot \mathbf{B} = \sum_{i=1}^{n} A_i B_i = A_1 B_1 + A_2 B_2 + \cdots + A_n B_n$$

Dot product



Convolution



3. Simulation

3.3 Phase Correlation

- Compute Cross power spectrum of F1 and F2

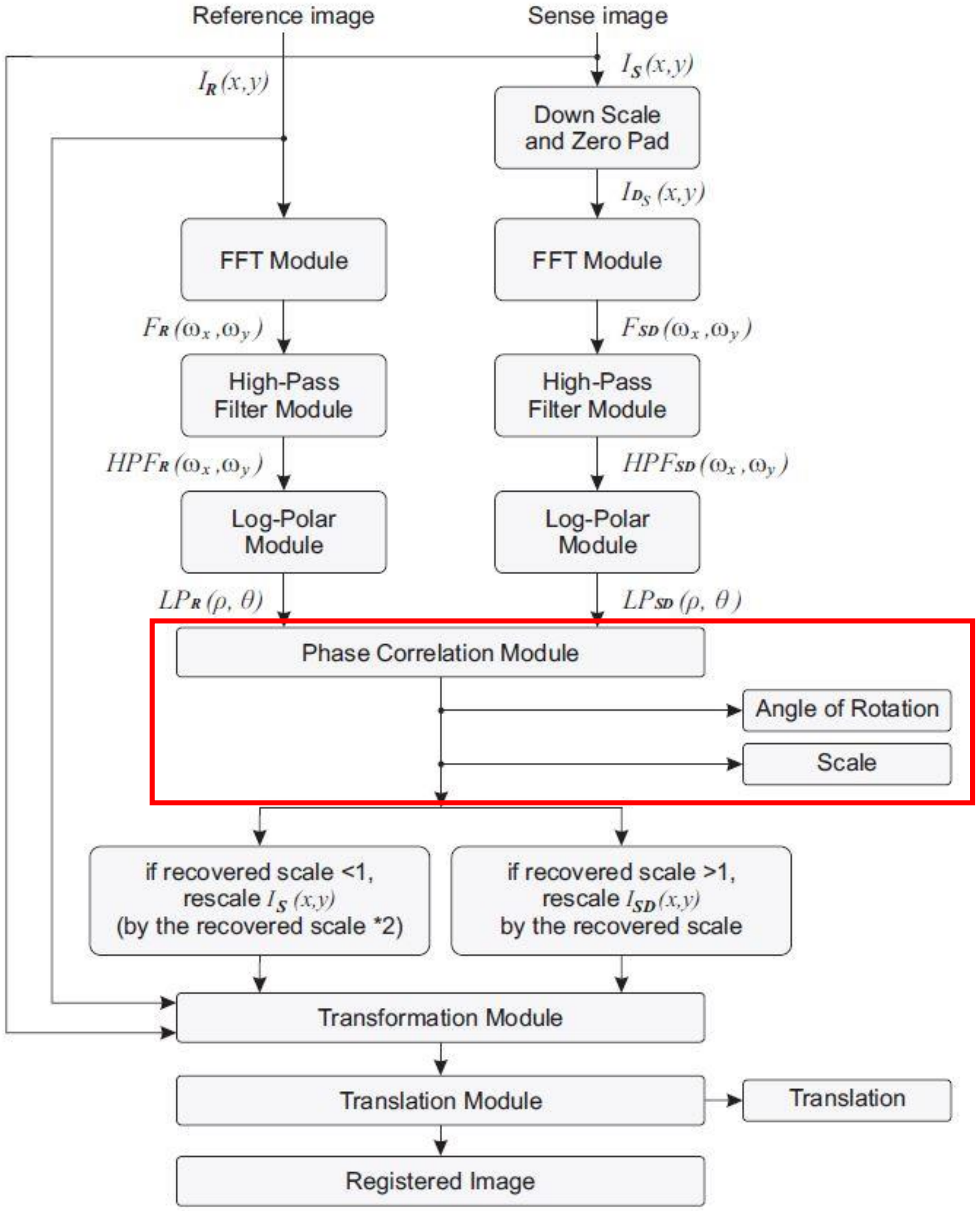
$$\frac{F(\xi,\eta)F'^{*}(\xi,\eta)}{|F(\xi,\eta)F'(\xi,\eta)|} = e^{j2\pi(\xi x_0+\eta y_0)}$$

※ Phase of the cross-power spectrum is equivalent to the phase difference between the images.

⇩

Find the location in images

of the peak of the phase correlation



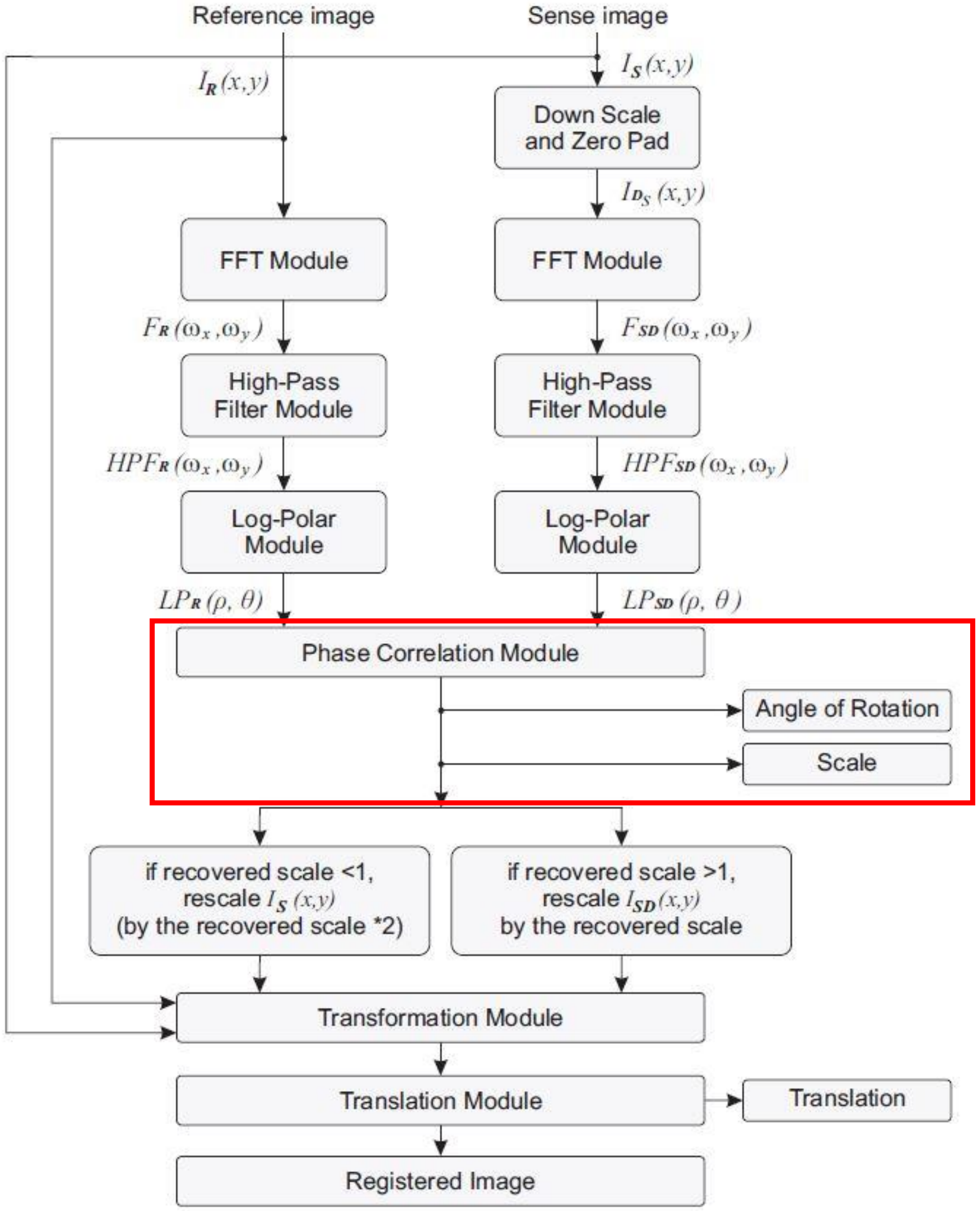
3. Simulation

3.4 Compute angle of rotation

$$F(u,v) = |F(u,v)|e^{j\theta(u,v)}$$

At, $|F(u,v)|$ *is Fouier spectrum and* $\theta(u,v)$ is phase spectrum

$\theta_1 - \theta_2$ = Angle of rotation



3. Simulation

3.5 Transformation and Translation

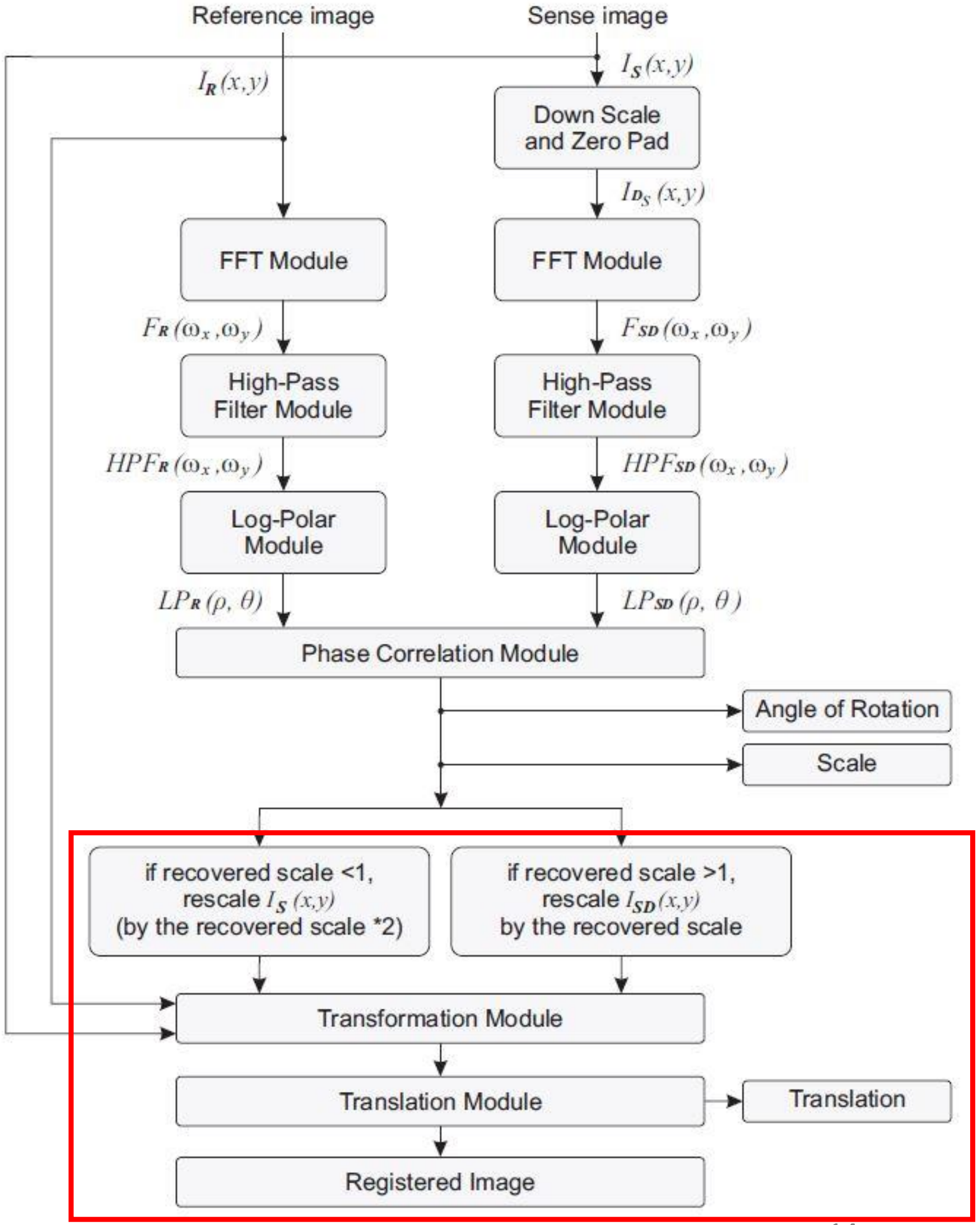
Recovered scale

&

de-rotate Sense image

- Reference image

- Final image.



Future Plan