We will start to presentation soon!

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Orbit Determination Using Lambert Problem Based on Newton-Raphson Method

1.

Review 'Day-Three'

We know...

- ✗ Two position vectors (r_1, r_2)
- ✗ One interval time (Δt)

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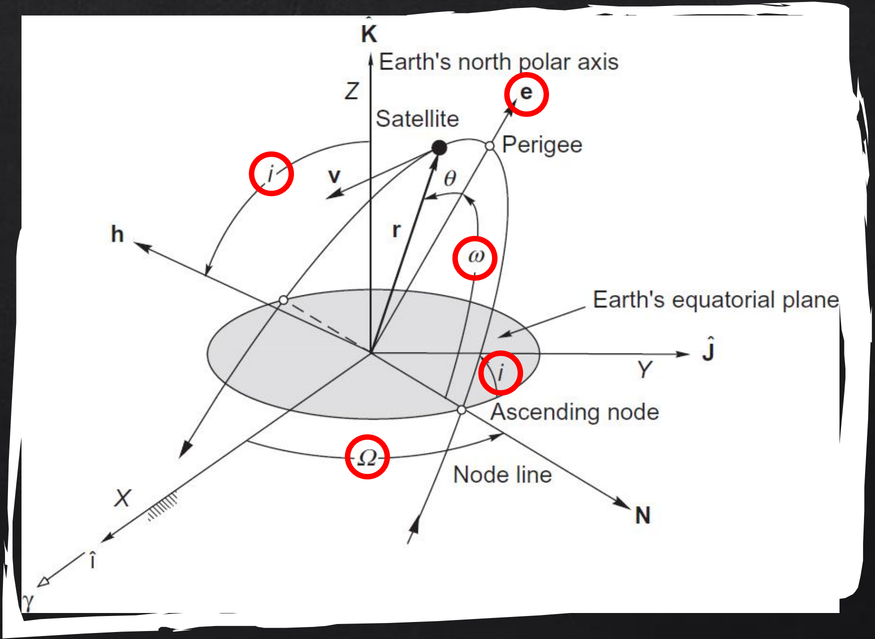
For orbit determination



Orbital Six Elements

For orbital six elements

We must find velocity vector (v)

$$v_1 = \frac{1}{g}\left(r_2 - f r_1\right)$$

For velocity vector

$$f = 1 - \frac{r_2}{p}\left(1 - \cos\Delta\theta\right) = 1 - \frac{a}{r_1}\left(1 - \cos\Delta E\right)$$

$$g = \frac{r_1 r_2 \sin\Delta\theta}{\sqrt{\mu p}} = \Delta t - \sqrt{\frac{a^3}{\mu}}\left(\Delta E - \sin\Delta E\right)$$

$$\dot{f} = \sqrt{\frac{\mu}{p}}\tan\frac{\Delta\theta}{2}\left(\frac{1 - \cos\Delta\theta}{p} - \frac{1}{r_1} - \frac{1}{r_2}\right) = -\frac{\sqrt{\mu a}}{r_1 r_2}\sin\Delta E$$

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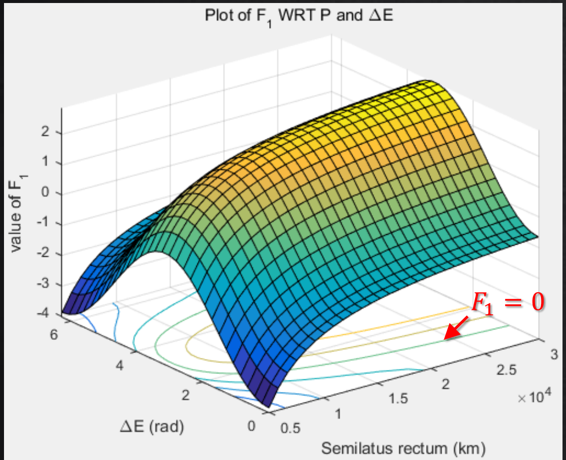
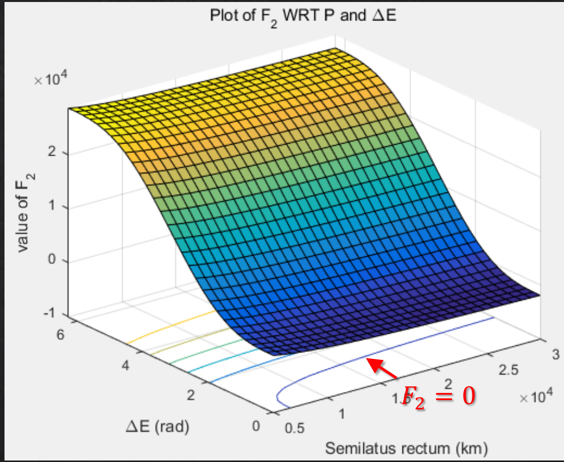
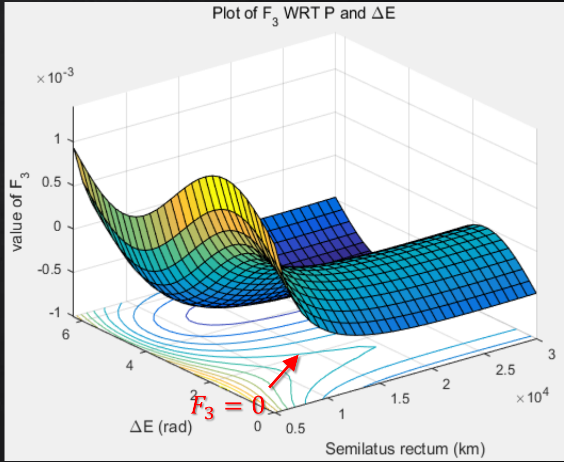
2.

Neton-Raphson Method

Illustrative examples

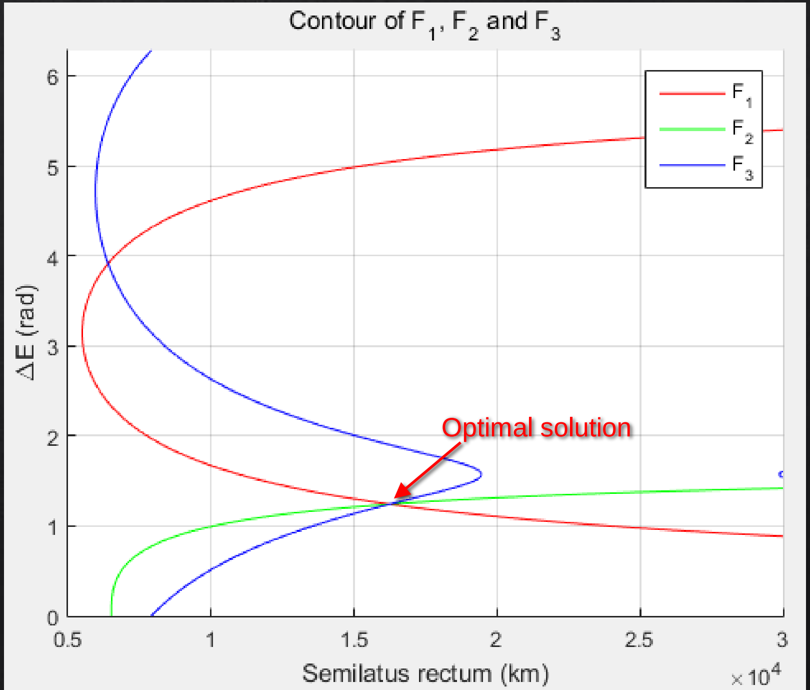






Plot of F with respect to P and E

Contour of functions



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At nonlinear systems of equations...

$$x_{n+1} = x_n - J\left(x_n\right)^{-1} F(x_n)$$

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At the Newton-Raphson method...

$$x_{n+1} = x_n - J(x_n)^{-1} F(x_n)$$

$$x = \begin{bmatrix} a \\ p \\ \Delta E \end{bmatrix}, \qquad J = \frac{\partial F}{\partial x}, \qquad F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

F IS...

$$F = \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix}^T$$

$$F_1 = \frac{a}{r_1}\left(1 - \cos\Delta E\right) - \frac{r_2}{p}\left(1 - \cos\Delta\theta\right) = 0$$

$$F_2 = \sqrt{\frac{a^3}{\mu}}\left(\Delta E - \sin\Delta E\right) + \frac{r_1 r_2 \sin\Delta\theta}{\sqrt{\mu p}} - \Delta t = 0$$

$$F_3 = \frac{\sqrt{\mu a}}{r_1 r_2}\sin\Delta E + \sqrt{\frac{\mu}{p}}\tan\frac{\Delta\theta}{2}\left(\frac{1 - \cos\Delta\theta}{p} - \frac{1}{r_1} - \frac{1}{r_2}\right) = 0$$

J is...

$$J = \frac{\partial F}{\partial x} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} \end{bmatrix}$$

Thus,

$$x_{n+1} = x_n - J\left(x_n\right)^{-1} F(x_n)$$

$$\begin{bmatrix} a_{n+1} \\ p_{n+1} \\ \Delta E_{n+1} \end{bmatrix} = \begin{bmatrix} a_n \\ p_n \\ \Delta E_n \end{bmatrix} - \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} \end{bmatrix}^{-1} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

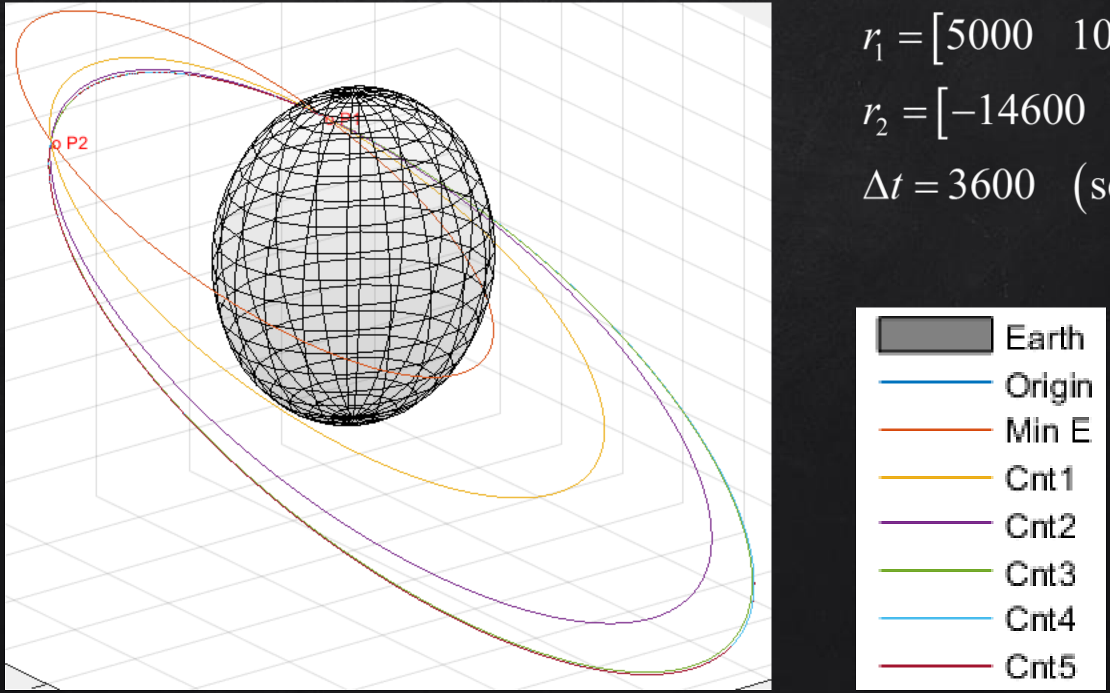
3.

Orbit Determination

Initial values...

$$x_0 = \begin{bmatrix} a_{\min} \\ p_{\min} \\ \Delta\theta \end{bmatrix}$$

Orbit shape...

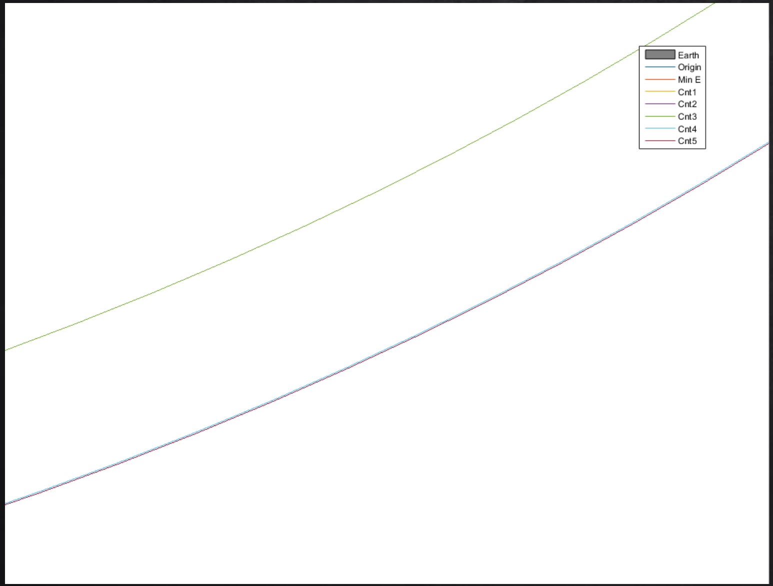


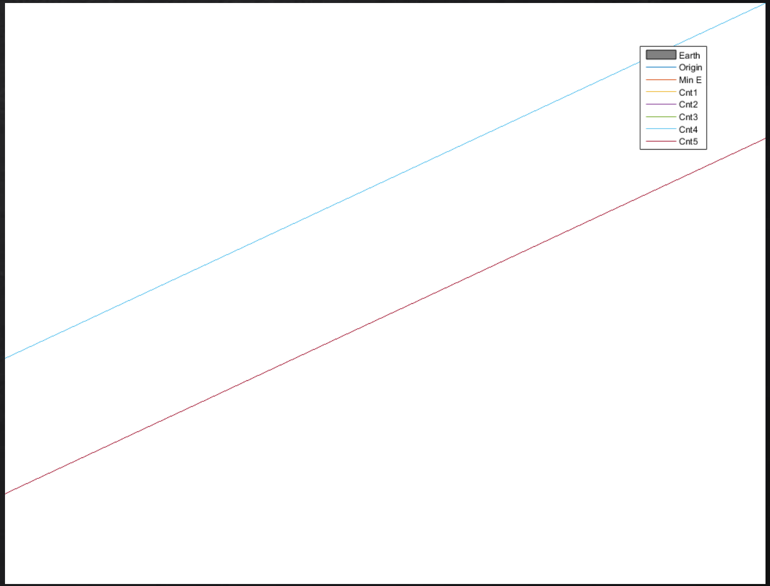
$$r_1 = [5000 \quad 10000 \quad 2100] \quad (km)$$

$$r_2 = [-14600 \quad 2500 \quad 7000] \quad (km)$$

$$\Delta t = 3600 \quad (\text{sec})$$

Zoom in





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4.

And!

Thanks!