

2015

Daythree Lab Seminar Controla

**A Low-complexity global
approximation free control scheme
with prescribed performance function.**

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The master's course
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I. Concept of GAFC.

- I. Nonlinear control.
- II. Error.
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- I. Demonstration.
- II. Dynamics Simulation using Matlab.

1. Concept of A low- Complexity Global Approximation free Control

I . Concept of GAFC.

1. Nonlinear Control

A Low complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems.

- Automatica, C.P. Bechlioulis, G.A. Rovithakis. 2014.

Why use nonlinear control?

- Real systems are nonlinear.
- High Accuracy.
- Unnecessary for Linearization.
- But it has **Complex Controller**.

What is the advantage of GAFC?

- **Simple Controller.**
- Looks like Linear controller.
- 'P-like' controller type.

$$u = g(x)^{-1} \left\{ -f(x) + \dot{x}_d - ke - c_1 s - c_2 \text{sat}(s) \right\}$$

$$u = -k_p e$$

I . Concept of GAFC.

2. Error Definition.

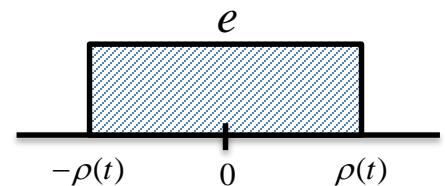
error

$$e = x - x_d$$

$$\frac{x - x_d}{\rho(t)} = \frac{e}{\rho(t)} = Z \quad (\text{where, if } \rho(t) > |e|)$$

⇓

$$-\rho(t) < e < \rho(t) \Rightarrow$$



Therefore,

$$-1 < \frac{x - x_d}{\rho} < 1 \quad -1 < Z < 1$$

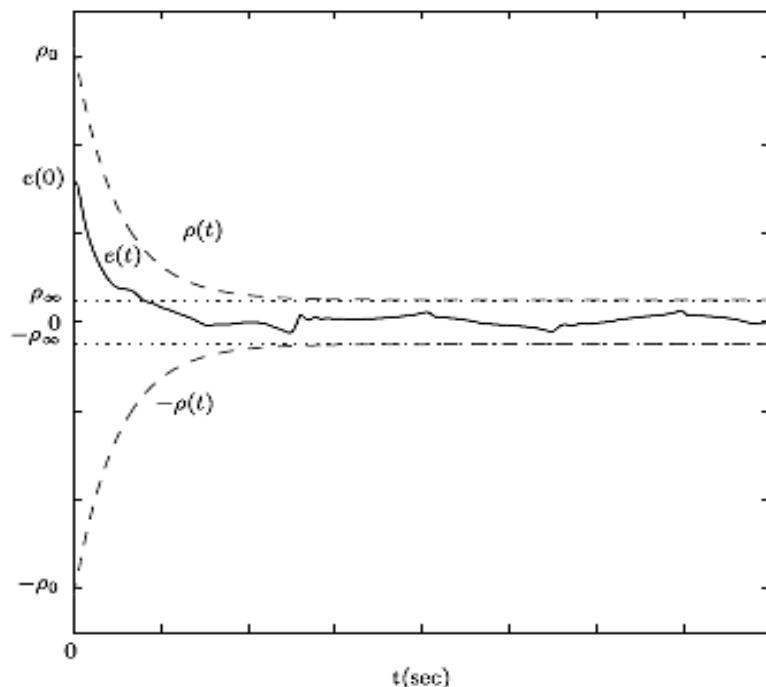
I . Concept of GAFC.

3. Prescribe Performance function.

$$\rho(t) = (\rho_0 - \rho_\infty)e^{-at} + \rho_\infty$$

(where, ρ_0 = Initial value.
 ρ_∞ = Allowable error value.
 a = Decreasing rate.

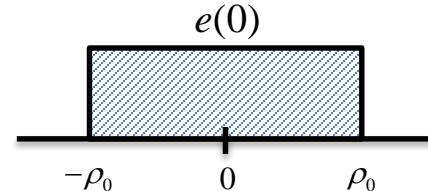
they are strictly positive constants.)



$$\rho_0 > |e(0)|$$



$$-\rho_0 < e(0) < \rho_0$$



I . Concept of GAFC.

4. Error Dynamics Definition.

$$\varepsilon(t) = \ln\left(\frac{1+Z}{1-Z}\right) \iff e^{\varepsilon(t)} = \frac{1+Z}{1-Z} \quad \left(\text{where, } \frac{1+Z}{1-Z} > 0 \rightarrow -1 < Z < 1 \right)$$

$$\iff (1-Z) e^{\varepsilon(t)} = 1 + Z$$

$$\iff (1+e^{\varepsilon(t)})Z = e^{\varepsilon(t)} - 1$$

$$\iff Z = \frac{e^{\varepsilon(t)} - 1}{e^{\varepsilon(t)} + 1}$$

$$\iff Z = \frac{e^{\frac{\varepsilon(t)}{2}} - e^{-\frac{\varepsilon(t)}{2}}}{e^{\frac{\varepsilon(t)}{2}} + e^{-\frac{\varepsilon(t)}{2}}} \quad \iff Z = \tanh \frac{\varepsilon(t)}{2}$$

I . Concept of GAFC.

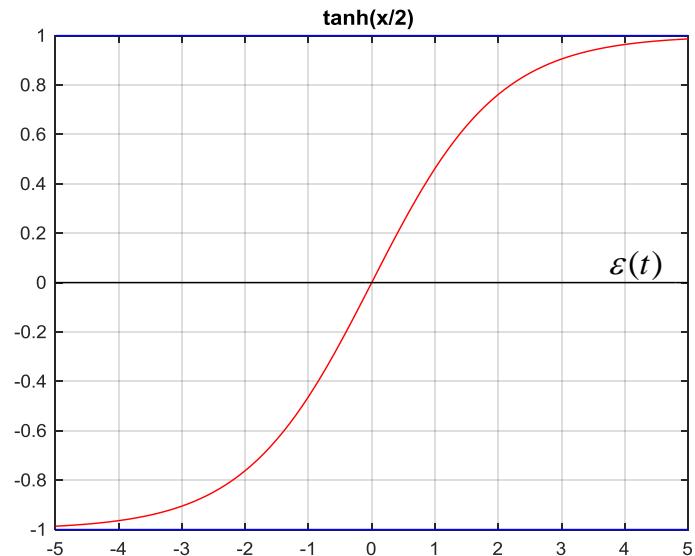
4. Error Dynamics Definition.

$$\varepsilon(t) = \ln\left(\frac{1+Z}{1-Z}\right) \Leftrightarrow Z = \tanh\frac{\varepsilon(t)}{2}$$

$$-\infty < \varepsilon(t) < \infty$$

↓

$$-1 < Z < 1$$



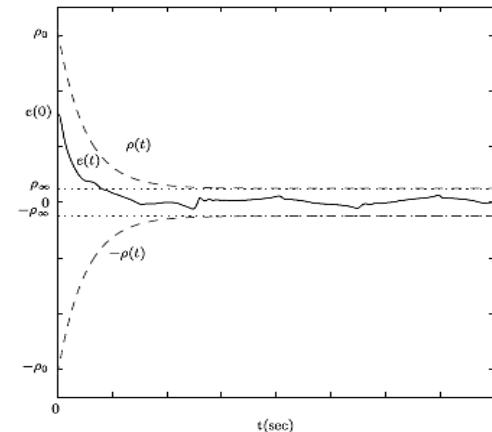
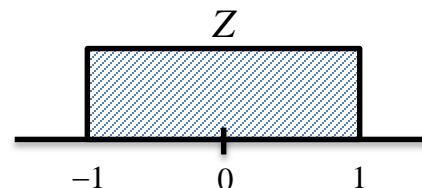
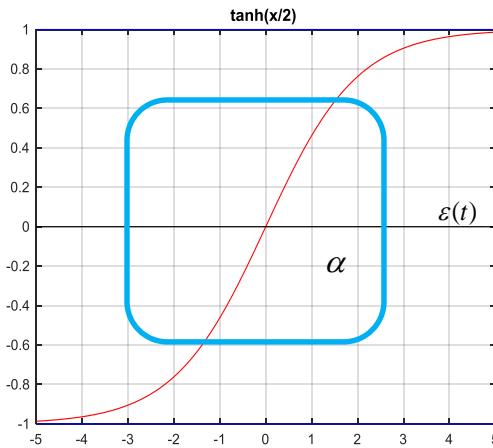
I . Concept of GAFC.

5. Relation of Equation.

$$Z = \tanh \frac{\varepsilon(t)}{2}$$

$$Z = \frac{x - x_d}{\rho}$$

$$-\alpha < \varepsilon(t) < \alpha \quad \Rightarrow \quad -1 < Z < 1 \quad \Rightarrow \quad -\rho(t) < e(t) < \rho(t)$$



2. Demonstration & Simulation.

II. Demonstration & Simulation.

1. Demonstration.

Lyapunov stability

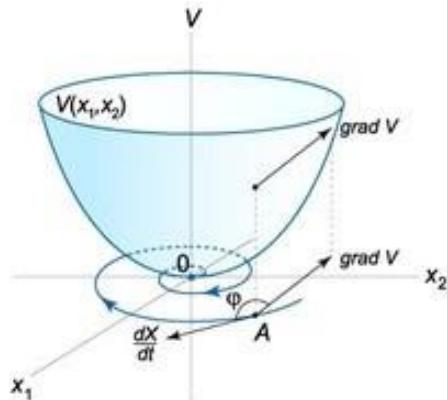
$$f(x) = V = \frac{1}{2}\varepsilon^2 \quad (\varepsilon \neq 0, V > 0)$$

$$\text{if, } \exists \quad f(x)' = \dot{V} = -K^* \varepsilon^2 \quad (K^* > 0, \varepsilon \neq 0, \dot{V} < 0)$$

$$\dot{V} = -2K^* V \quad (2V = \varepsilon^2)$$

$$V = e^{-2K^* t} \quad (t \rightarrow \infty, V \rightarrow 0)$$

$$V = \frac{1}{2}\varepsilon^2 \quad (V \rightarrow 0, \varepsilon \rightarrow 0)$$



Hence, $V > 0$ & $\dot{V} < 0$: Asymptotically stable

II. Demonstration & Simulation.

1. Demonstration.

Lyapunov function $V = \frac{1}{2}\varepsilon^2 \quad (\varepsilon \neq 0, V > 0) \quad Z = \frac{x - x_d}{\rho(t)}, \quad \varepsilon(t) = \ln\left(\frac{1+Z}{1-Z}\right)$

$$\begin{aligned} \dot{V} &= \varepsilon \cdot \dot{\varepsilon} = \varepsilon \cdot \frac{2\dot{Z}}{1-Z^2} = \varepsilon \cdot \frac{2}{1-Z^2} \cdot \frac{1}{\rho} \cdot (f(x) - \dot{x}_d - z \cdot \dot{\rho} + u) \\ &= \varepsilon \cdot \frac{2}{1-Z^2} \cdot \frac{1}{\rho} \cdot (f(x) - \dot{x}_d - z \cdot \dot{\rho} - K_p \varepsilon(t)) \\ &\leq \frac{2}{1-Z^2} \cdot \frac{1}{\rho} \cdot (|f(x) - \dot{x}_d - z \cdot \dot{\rho}| \cdot |\varepsilon(t)| - K_p |\varepsilon(t)|^2) \end{aligned}$$

$$|f(x) - \dot{x}_d - z \cdot \dot{\rho}| \leq C_1$$

$$\dot{V} \leq \frac{2}{1-Z^2} \cdot \frac{1}{\rho} \cdot |\varepsilon(t)| (C_1 - K_p |\varepsilon(t)|)$$

$$(C_1 - K_p |\varepsilon(t)|) < 0$$

$$|\varepsilon(t)| > \frac{C_1}{K_p}$$

II. Demonstration & Simulation.

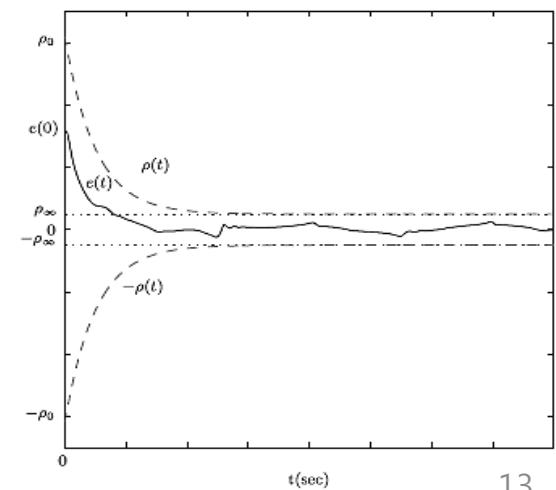
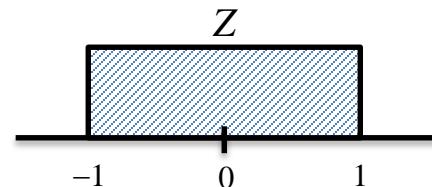
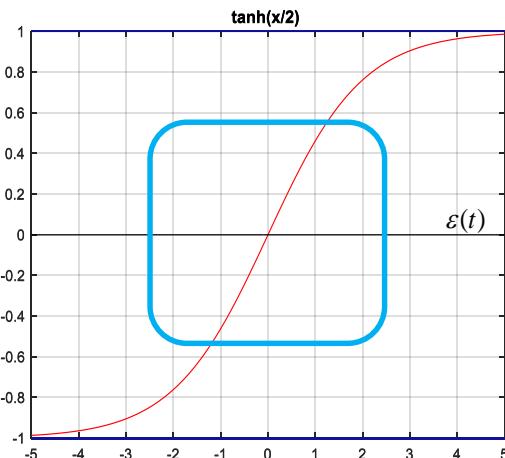
Lyapunov theory \rightarrow $V = e^{-2K^*t}$ $(t \rightarrow \infty, V \rightarrow 0)$ $V = \frac{1}{2}\varepsilon^2$ $(V \rightarrow 0, \varepsilon \rightarrow 0)$

Stability proof

$$Z = \tanh \frac{\varepsilon(t)}{2}$$

$$Z = \frac{x - x_d}{\rho}$$

$$-\alpha < \varepsilon(t) < \alpha \Rightarrow -1 < Z < 1 \Rightarrow -\rho(t) < e(t) < \rho(t)$$



II. Demonstration & Simulation.

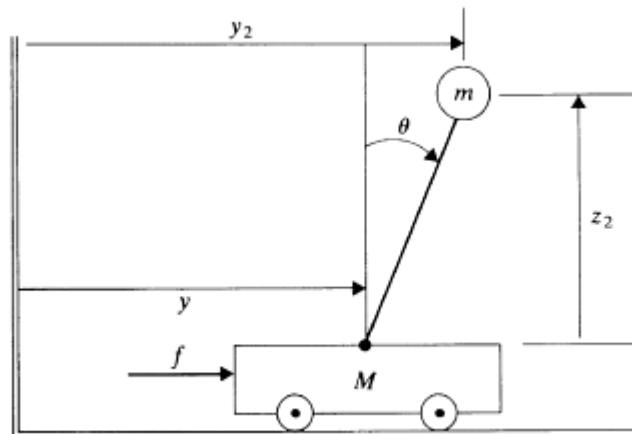
2. Simulation.

Dynamics

$$\dot{X} = f(x) + u$$

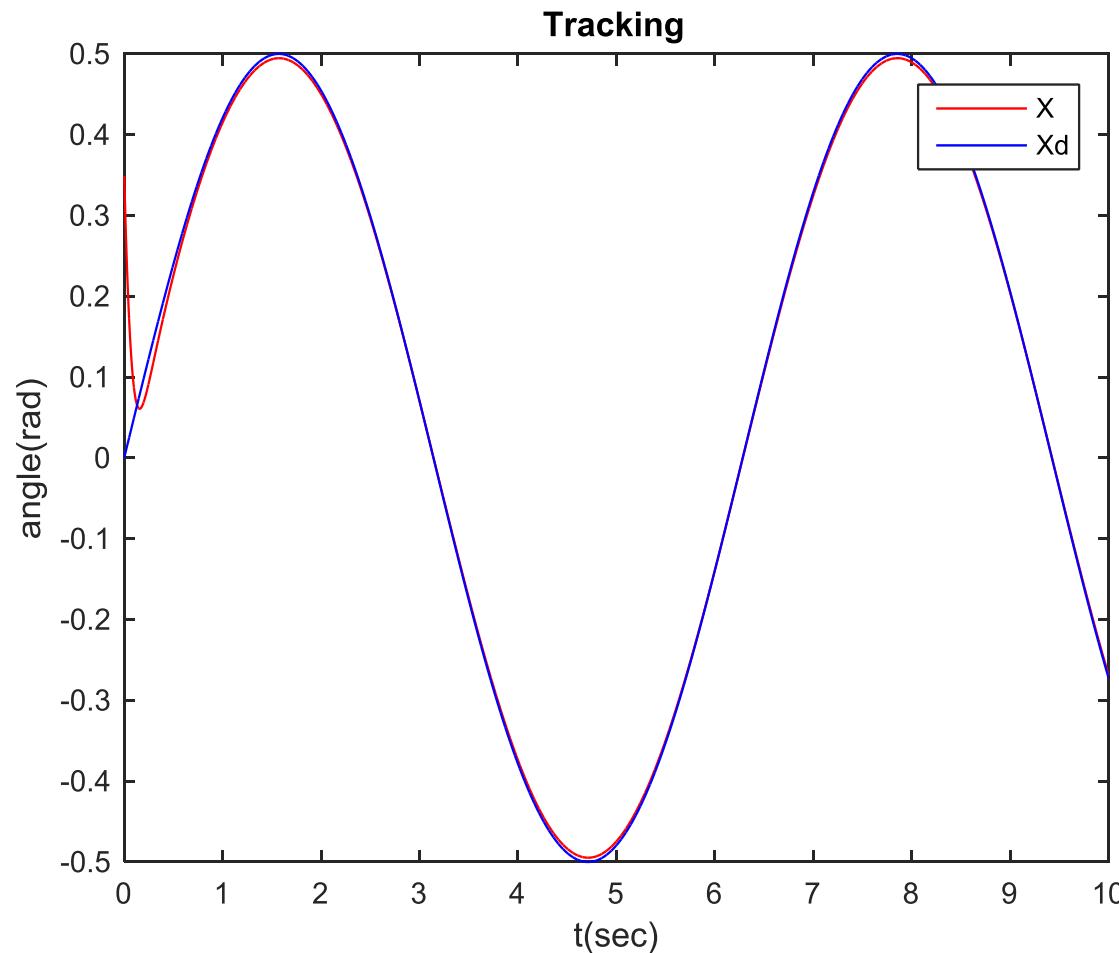
$$f(x) = \frac{m \cdot l \cdot x^2 + 2 \sin(x) \cdot \cos(x) - (M+m) \cdot g \cdot \sin(x)}{(m \cdot \cos(x)^2 + M+m)}$$

$$u(x) = -k \cdot \varepsilon(t)$$



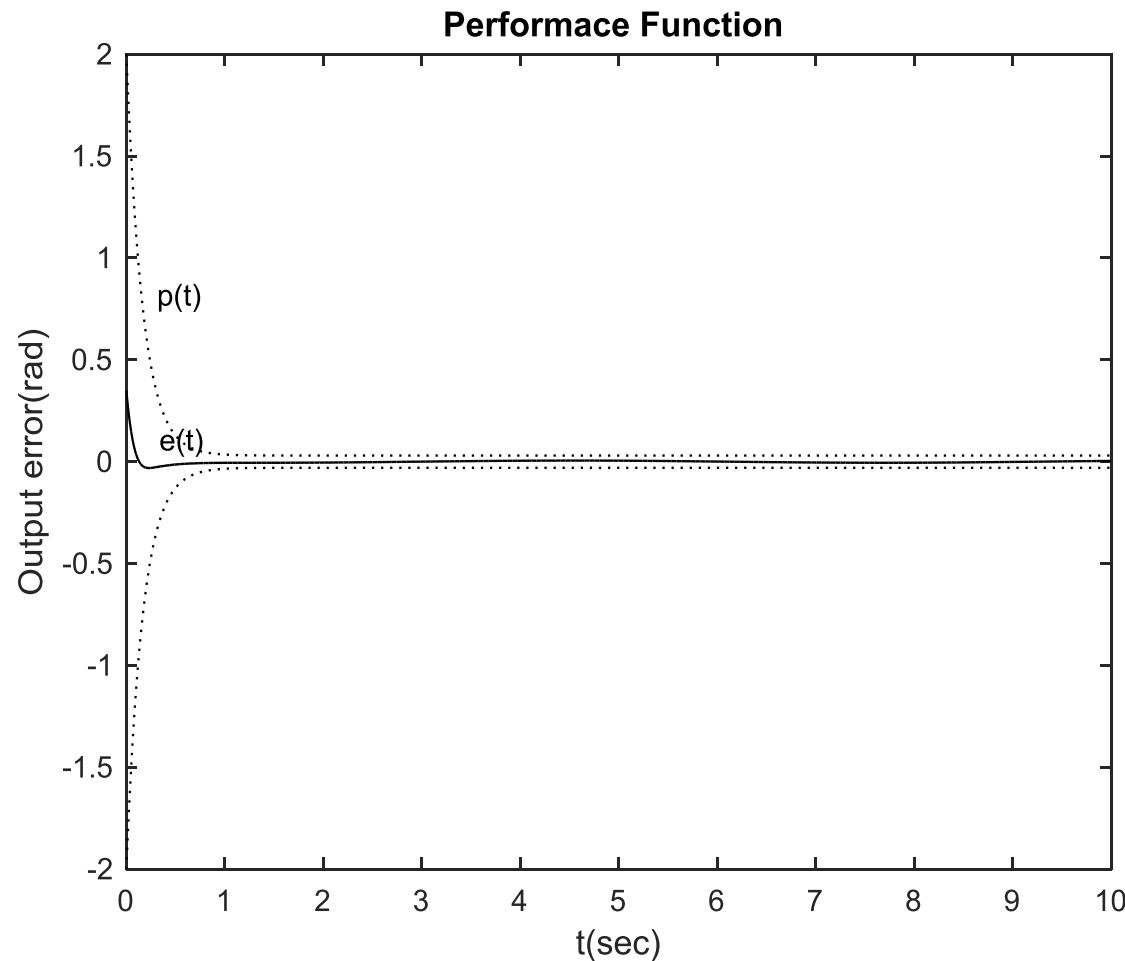
II. Demonstration & Simulation.

2. Simulation.



II. Demonstration & Simulation.

2. Simulation.



Future Plan.

- 1. Apply to a Quadcopter dynamics.**

- 2. Quadcopter Hovering.**