Spacecraft Attitude Control Using Quaternion Feedback

Pasion.Kim

2015 Controla Daytwo

Lab. For Navigation Control, and Applications

CONTROLA

Contents

I. Introduction.

II. Body.

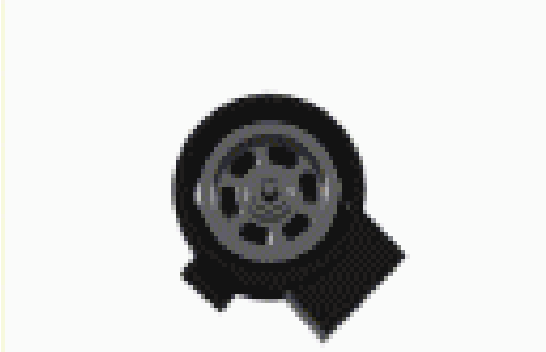
III. Simulation.

IV. Conclusion.

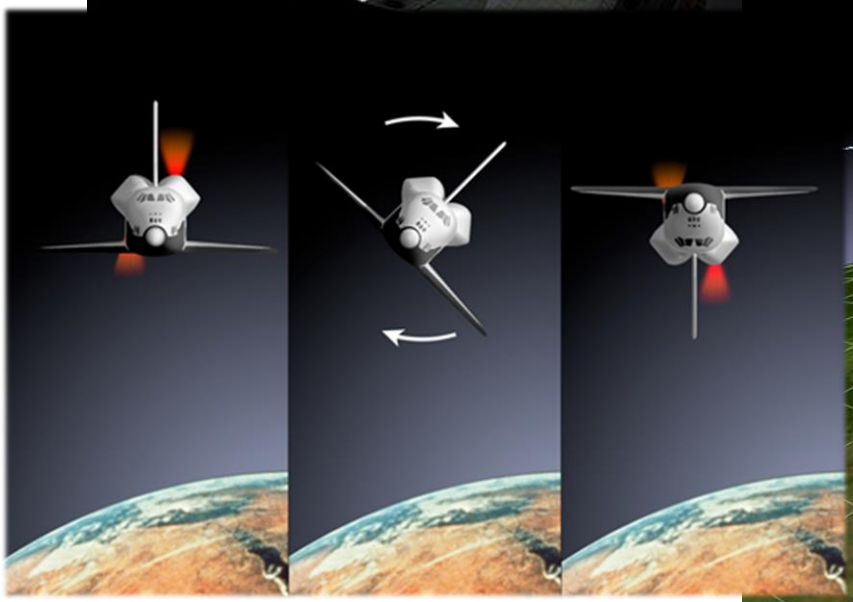
V. Future plan.

Ⅰ. Introduction(1/3)

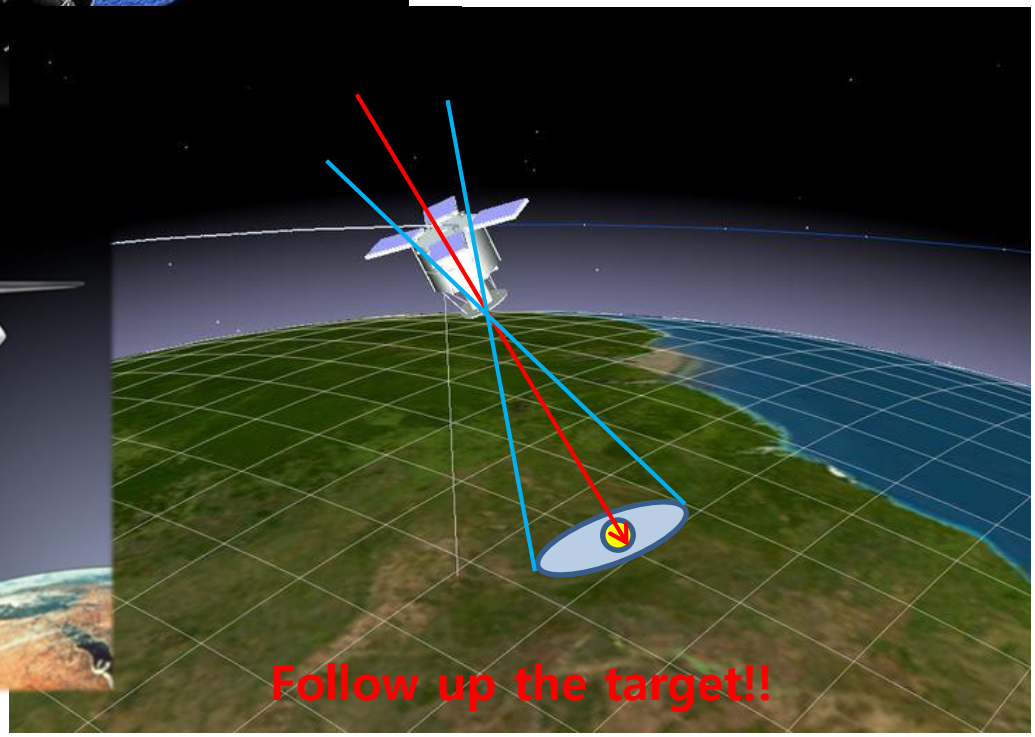
Q. Why have to control the attitude?



Reaction wheel

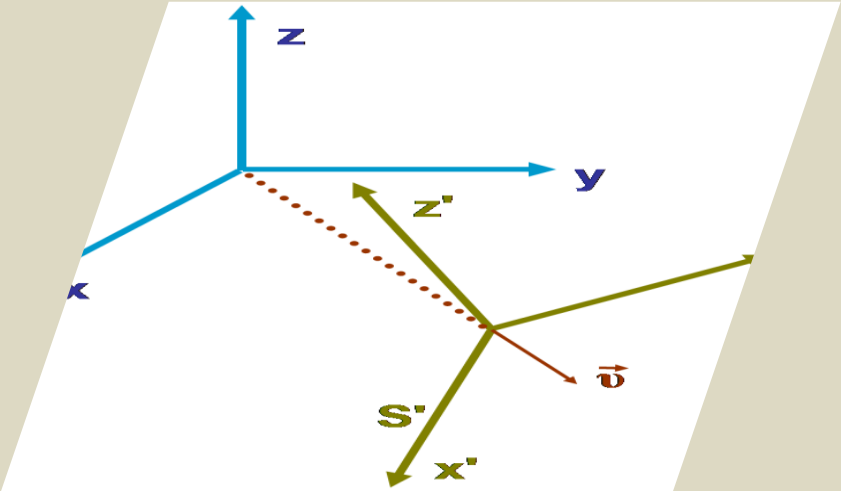


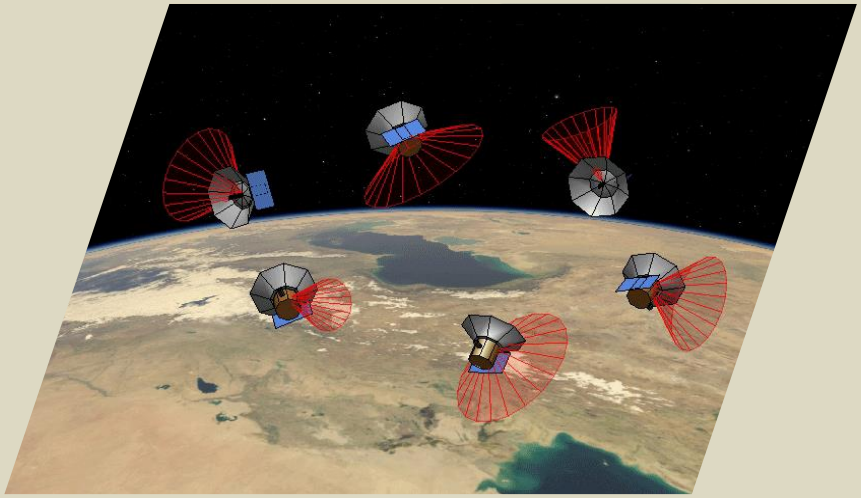
Thrust



Ⅰ. Introduction(2/3)

- **Method to obtain a Spacecraft attitude.**





Kinematics

- Coordinate, Reference frame.
- DCM, Euler, Quaternion, Euler's Eigenaxis.

Dynamics

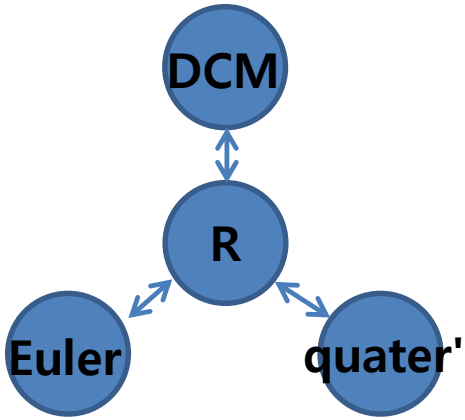
- Equation of the motion.
- Newton Method, Lagrange Method.

Ⅰ. Introduction(3/3)

- **Method of research for Spacecraft attitude control.**

Coordinate Transformation

- **Kinematics**
  - Direction Cosine Matrix
  - Euler Angle
  - Quaternion

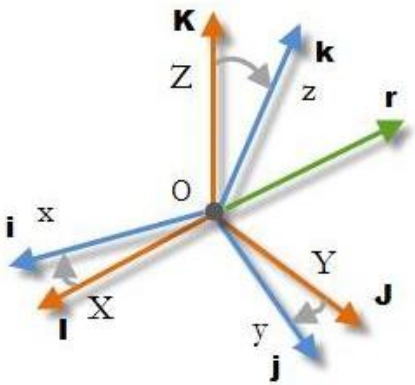


Attitude Control

- **Rotational Dynamics**
  - Newton Method
    (Euler's rotational Equation)
- **Controller**
  - Quaternion Feedback

Ⅱ. Body.(1/14)

1. Coordinate Transformation



$$\mathbf{r} = r_i\mathbf{i} + r_j\mathbf{j} + r_k\mathbf{k}$$
$$= r_I\mathbf{I} + r_J\mathbf{J} + r_K\mathbf{K}$$

- 3x3 방향코사인 행렬. (9개의 파라미터)
- 여러 단계의 연속적인 좌표축 변환 시 편리.
- 두 좌표 사이의 방향코사인 값으로 구성.

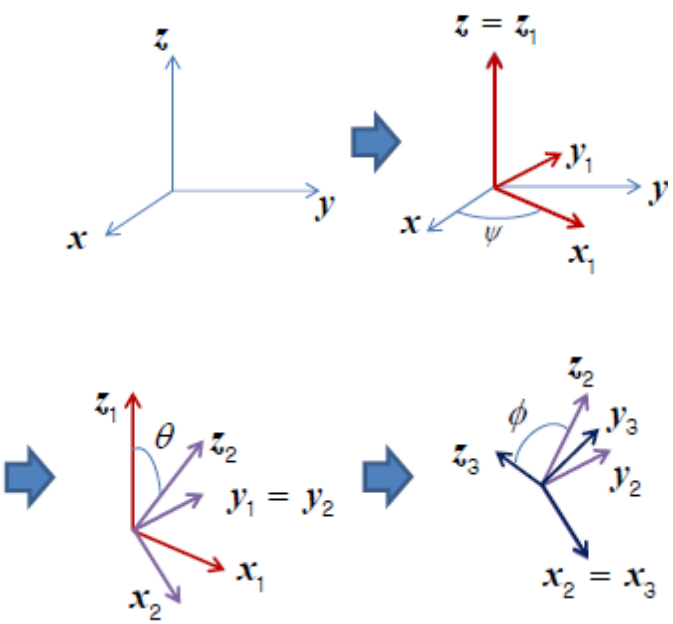
(- 즉, 두 좌표의 단위벡터의 내적으로 표현가능.)

$$C = \underbrace{\begin{pmatrix} x_2 \cdot x_1 & x_2 \cdot y_1 & x_2 \cdot z_1 \\ y_2 \cdot x_1 & y_2 \cdot y_1 & y_2 \cdot z_1 \\ z_2 \cdot x_1 & z_2 \cdot y_1 & z_2 \cdot z_1 \end{pmatrix}}_{\textit{DCM (DCM)}} = \underbrace{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}}_{\textit{R : transformation}}$$

$$\mathbf{r}_2 = C\mathbf{r}_1$$

Ⅱ. Body.(2/14)

1. Coordinate Transformation



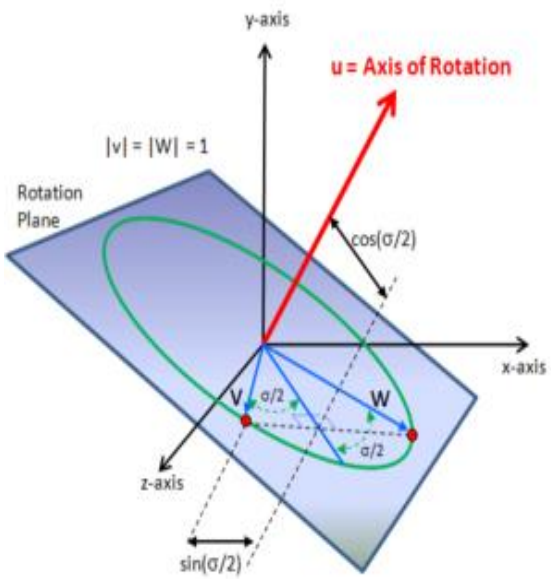
- 두 좌표계 사이의 회전을 나타내는 각. (ϕ, θ, ψ)
- 기하학적으로 정의가 명확하여 시각화에 용이.
- 큰 자세각에서 특이점에 빠질 수 있다.

$$C = \underbrace{\begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix}}_{\textbf{DCM (Euler)}} = \underbrace{\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}}_{\textbf{R}}$$

$$\mathbf{x}_2 = [C = C^{321} = C(\phi) * C(\theta) * C(\psi)] * \mathbf{x}_1$$

Ⅱ. Body.(3/14)

1. Coordinate Transformation



- 정의 : 두 자세간의 기동은 한번의 회전으로 변환할 수 있다.
  (오일러 회전이론, 변수 : 4개 (오일러 축, 오일러 주각))
- 계산과정이 간단하여 탑재 실시간 계산에 적합.
- 큰 자세각에서 특이점 없이 모든 자세 표현가능.

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = \begin{bmatrix} \|\vec{e}\| \cdot \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{bmatrix}$$

$$C = \begin{pmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

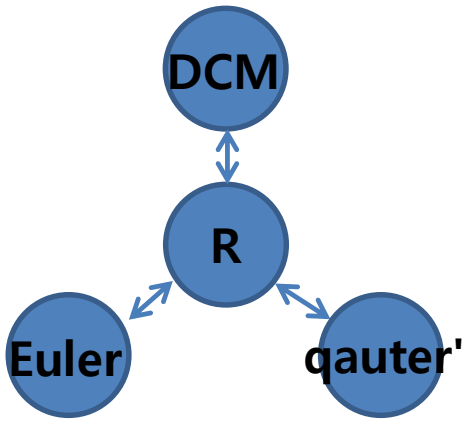
DCM (quaternion) ***R***

Ⅱ. Body.(4/14)

1. Coordinate Transformation

Kinematics

Relationships with rotation matrix
(R ←→ DCM, Euler , Quaternion)



$$R_1^2 = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

DCM →R→ Euler :

$$\phi = \tan^{-1}\frac{c_{23}}{c_{33}} \qquad \theta = -\sin^{-1} c_{13} \qquad \psi = \tan^{-1}\frac{c_{12}}{c_{11}}$$

DCM →R→ $quat$:

$$q_4 = 0.5\sqrt{1 + c_{11} + c_{22} + c_{33}} \qquad q_2 = \frac{0.25(c_{31} - c_{13})}{q_4}$$

$$q_1 = \frac{0.25(c_{23} - c_{32})}{q_4} \qquad q_3 = \frac{0.25(c_{12} - c_{21})}{q_4}$$

Euler →R→ $quat$:

$$\theta_x = \tan^{-1}\left[\frac{2\,(q_2 q_3 + q_0 q_1)}{q_0^2 - q_1^2 - q_2^2 + q_3^2}\right], -\pi \le \theta_x < \pi$$

$$\theta_y = -\sin^{-1}(2q_1 q_3 - 2q_0 q_2), -\pi/2 \le \theta_y \le \pi/2$$

$$\theta_z = \tan^{-1}\left[\frac{2\,(q_1 q_2 + q_0 q_3)}{q_0^2 + q_1^2 - q_2^2 - q_3^2}\right], 0 \le \theta_z \le 2\pi$$

$quat$ →R→ $Euler$:

$$q_0 = \cos(\theta_z/2)\cos(\theta_y/2)\cos(\theta_x/2) + \sin(\theta_z/2)\sin(\theta_y/2)\sin(\theta_x/2)$$

$$q_1 = \cos(\theta_z/2)\cos(\theta_y/2)\sin(\theta_x/2) - \sin(\theta_z/2)\sin(\theta_y/2)\cos(\theta_x/2)$$

$$q_2 = \cos(\theta_z/2)\sin(\theta_y/2)\cos(\theta_x/2) + \sin(\theta_z/2)\cos(\theta_y/2)\sin(\theta_x/2)$$

$$q_3 = \sin(\theta_z/2)\cos(\theta_y/2)\cos(\theta_x/2) - \cos(\theta_z/2)\sin(\theta_y/2)\sin(\theta_x/2)$$

Ⅱ. Body.(5/14)

1. Coordinate Transformation

Kinematics

Compute Using Matlab
(quat → DCM → Euler →quat)

quat =

0.7133	0.1100	0.3141	0.6168

DCM =

0.7784	0.5445	0.3124
-0.2306	-0.2148	0.9490
0.5838	-0.8108	-0.0417

EulerAngle =

92.5139	-18.2023	34.9703

DCM =

0.7784	0.5445	0.3124
-0.2306	-0.2148	0.9490
0.5838	-0.8108	-0.0417

quat =

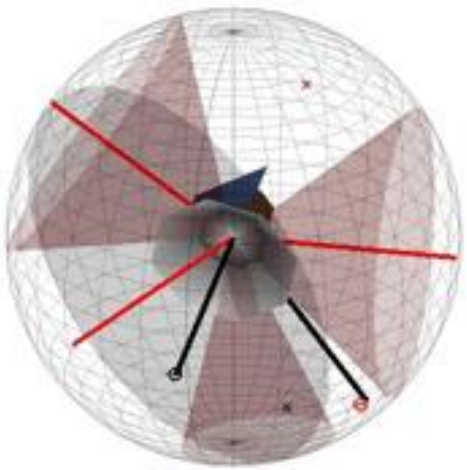
0.7133	0.1100	0.3141	0.6168

Ⅱ. Body.(6/14)

2. Attitude Control

Q. What is the dynamics?

A. Kinematics VS Dynamics



- **Kinematics(운동학)**
  - 물체가 운동하는 원인은 힘, 힘의 성질 해석 없이 어떻게 움직이는 지의 **운동의 상태만을 해석**.
  - 움직이는 물체의 효과를 알기 위해서 좌표계의 크기를 알아야 한다.
  - 좌표계의 크기를 **알고 있지 않은 채** 매 순간의 물체의 **속도**를 아는 것으로 해석 가능.
  - 운동의 크기를 나타내는 **운동량(Momentum)은 속도에 질량을 곱한 값**이다.

Ⅱ. Body.(7/14)

2. Attitude Control

Q. How can I turn?

A. Momentum.

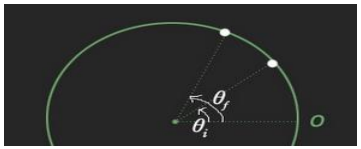
Ⅱ. Body.(8/14)

2. Attitude Control

- **Law of conservation of Momentum.**

- Motion : 1. 직선운동 – 물체의 방향이 그대로인 채 **위치만 변함**.

  2. 회전운동 – 위치는 그대로인 채 **방향만 변함**.



Rotating Motion VS Linear Motion (회전 운동과 선형 운동의 유사성)

	Rotating Motion (회전 운동)	Linear Motion (선형 운동)
변위 요소	θ	s
속도 요소	ω	v
가속도 요소	α	a
질량 요소	$\mathrm{I}(=mr^2)$	m
운동에너지	$\frac{1}{2}\mathrm{I}\omega^2$	$\frac{1}{2}mv^2$

	돌림힘(토크)	(선형 운동에서의) 힘
힘요소	$\mathrm{I}\alpha$ (관성모멘트)×(각가속도)	ma (질량)×(가속도)
변위 요소	θ	s
속도 요소	ω	v
가속도 요소	α	a
질량 요소	$\mathrm{I}(=mr^2)$	m

Ⅱ. Body.(9/14)

2. Attitude Control

- **Law of conservation of Momentum.**

Rotating Motion VS Linear Motion (회전 운동과 선형 운동의 유사성)

운동량 요소	각운동량 $L=I\omega$ (각운동량)=(관성모멘트)×(각가속도)	(선형 운동에서의) 운동량 $p=mv$ (운동량)=(질량)×(선속도)
변위 요소	θ	s
속도 요소	ω	v
가속도 요소	α	a
질량 요소	$I(=mr^2)$	m
운동에너지	$\frac{1}{2}I\omega^2$	$\frac{1}{2}mv^2$

Ⅱ. Body.(10/14)

2. Attitude Control

- **Law of conservation of Momentum.**

  - 운동량보존 : 1. 선운동량 – **선운동량의 시간의 변화가 힘**이다.

    – $P = mv,\ F = ma\ (\Sigma F = 0,\ \frac{dP}{dt} = F,\ \underline{p = const})$

    2. 각운동량 – **각운동량의 시간의 변화가 회전힘**이다.

    – $L = mr^2\omega,\ \tau = I\alpha\ (\Sigma\tau = 0,\ \frac{dL}{dt} = \tau,\ \underline{L = const})$

Ⅱ. Body.(11/14)

2. Attitude Control

- **Angular Momentum.**
  - 운동량보존 : 각운동량 – 각운동량의 시간의 변화가 회전힘이다.

$$- \ \Sigma\boldsymbol{\tau} = \dot{\mathbf{H}} = \frac{d\mathbf{H}}{dt} \ (where,\ \mathbf{H} = \mathbf{J}\boldsymbol{\omega})$$

$$= \mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} \ \ (Euler's\, rotational\, Equation)$$

  - 회전좌표축의 시간미분의 변환.
    (회전축과 비회전축 사이에서의 위치벡터의 시간에 대한 미분의 변환)

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

$$\dot{\mathbf{V}}_{XYZ} = (\dot{V}_x\mathbf{i} + \dot{V}_y\mathbf{j} + \dot{V}_z\mathbf{k}) + (V_x\dot{\mathbf{i}} + V_y\dot{\mathbf{j}} + V_z\dot{\mathbf{k}}) \ (total\, derivative)$$

$$\dot{\mathbf{V}}_{XYZ} = \dot{\mathbf{V}}_{xyz} + (\boldsymbol{\omega} \times V_x\mathbf{i} + \boldsymbol{\omega} \times V_y\mathbf{j} + \boldsymbol{\omega} \times V_z\mathbf{k})$$

$$\dot{\mathbf{V}}_{XYZ} = \dot{\mathbf{V}}_{xyz} + (\boldsymbol{\omega} \times \mathbf{V})$$

Ⅱ. Body.(12/14)

2. Attitude Control

- **Derivative of omega and quaternion according to input.**
  - 각운동량보존

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}(-\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\tau}) \qquad input: \begin{bmatrix} \tau \\ \mathbf{H}\ (CMG) \end{bmatrix}$$

  - 쿼터니언변환 (자세각)

$$\mathbf{u}(\tau) \rightarrow \dot{\omega} \rightarrow \omega \rightarrow \dot{q} \rightarrow q$$

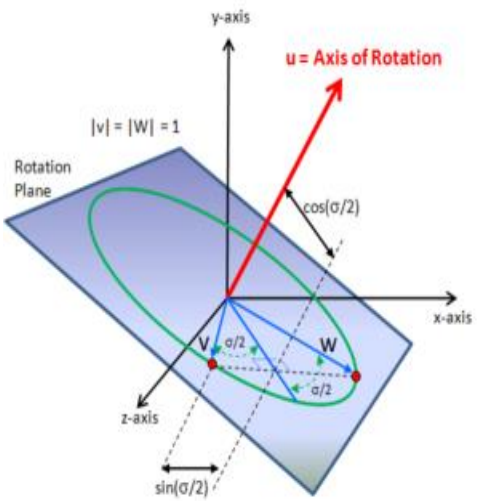
Input Plant Output

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \frac{1}{2}\boldsymbol{\Omega}' q$$

Ⅱ. Body.(13/14)

2. Attitude Control

- **Quaternion Feedback control.**



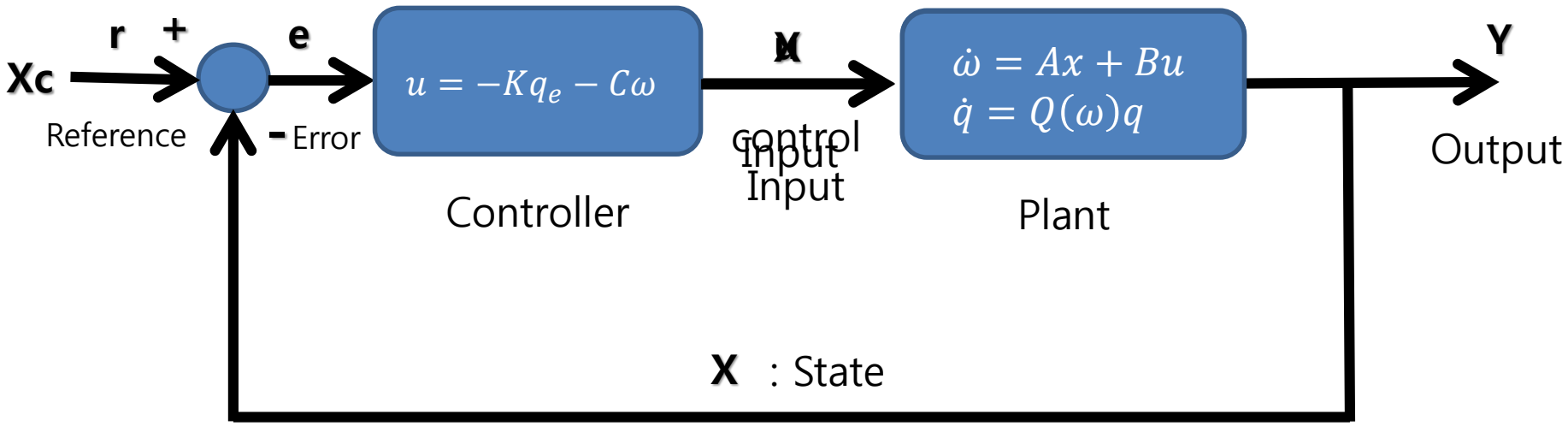
- 정의 : 두 자세간의 기동은 한번의 회전으로 변환할 수 있다.
  (오일러 회전이론, 변수 : 4개 (오일러 축, 오일러 주각))
- 계산과정이 간단하여 탑재 실시간 계산에 적합.
- 큰 자세각에서 특이점 없이 모든 자세 표현가능.

$$u = -Kq_e - C\omega$$

Ⅱ. Body.(14/14)

2. Attitude Control

Quaternion Feedback Control

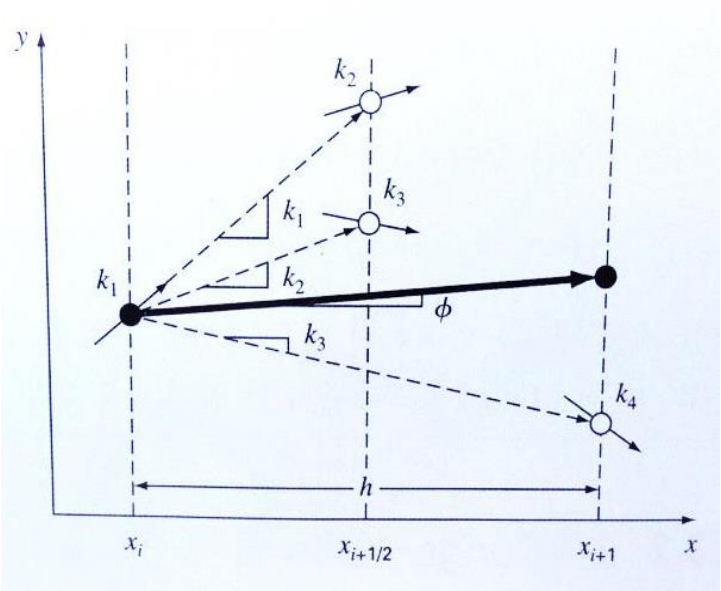


Xc : 목표 값, X : 상태 값, $q_e = Q(qc)q$

Ⅲ. Simulation.(1/2)

1. Spacecraft Attitude Control Using Quaternion Feedback Simulation.

Runge-Kutta (RK4)



Euler_c =

52
0
40

Euler_result =

52.0000
-0.0000
40.0000

quat_c =

0.4119
0.1499
0.3074
0.8446

quat_result =

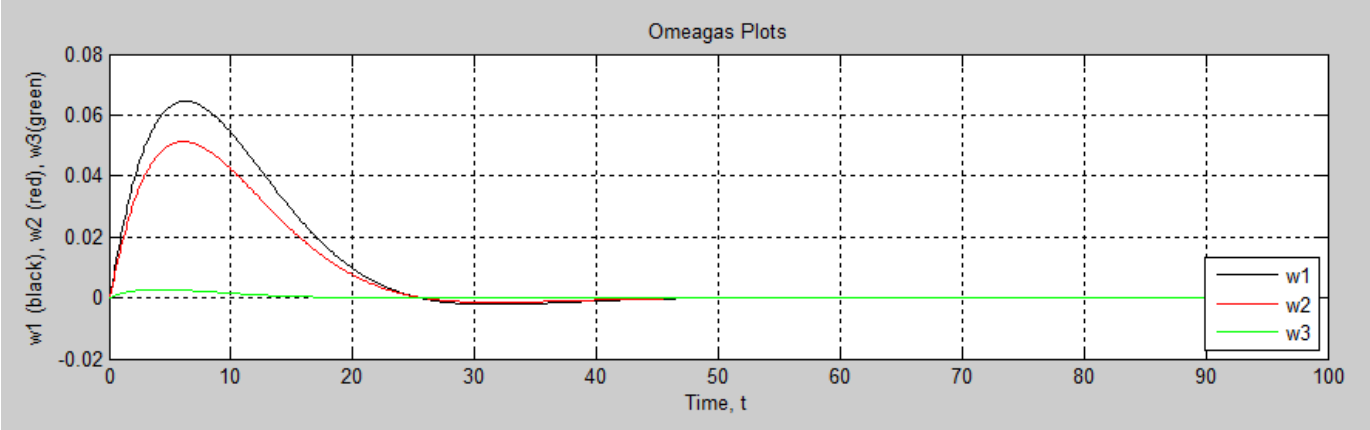
0.4119
0.1499
0.3074
0.8446

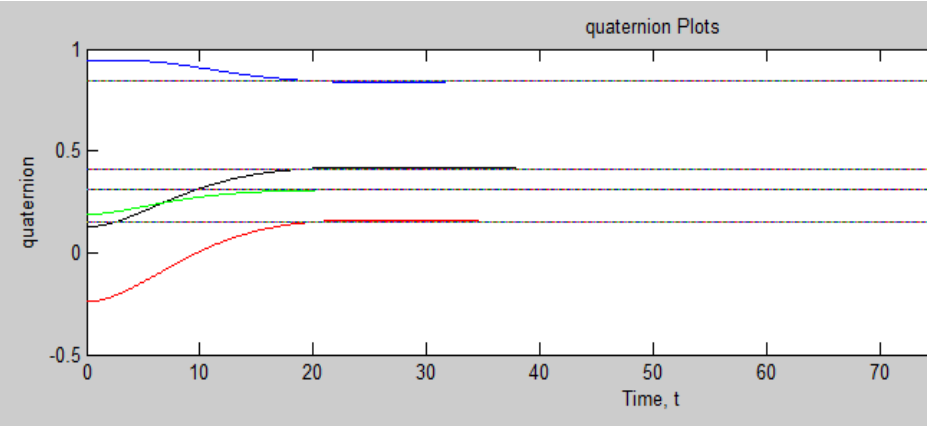
Ⅲ. Simulation.(2/2)

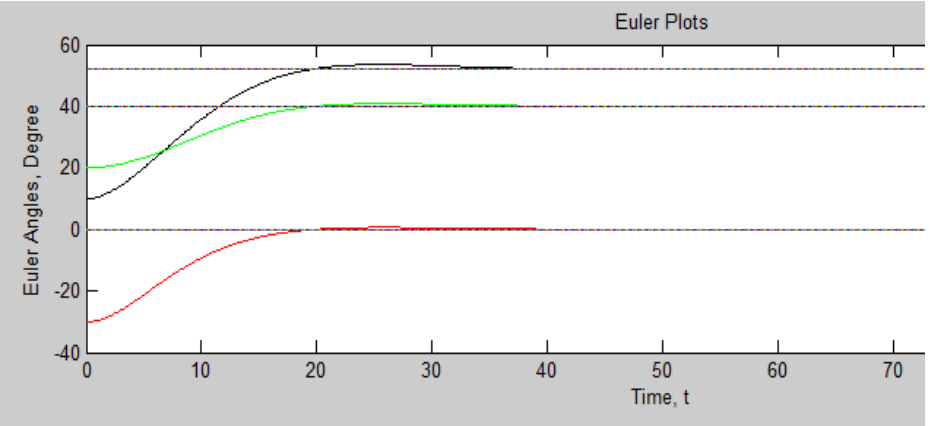
1. Spacecraft Attitude Control Using Quaternion Feedback Simulation.

$\Delta t : 0.1$
$t : 0 \sim 95sec$

$K : 1$
$C : 4$







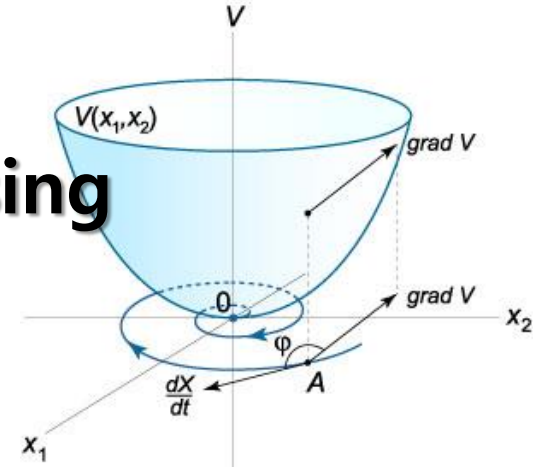
IV. Conclusion.(1/1)

1. 인공위성의 자세제어를 쿼터니언 피드백을 사용하여 시뮬레이션을 수행한 결과 오일러 각에 비해 큰 자세 각에서도 특이점 없이 정확하게 명령을 추종하는 것을 확인하였다.

2. 계산과정이 간단하여 탑재 실시간 계산에 적합하다.

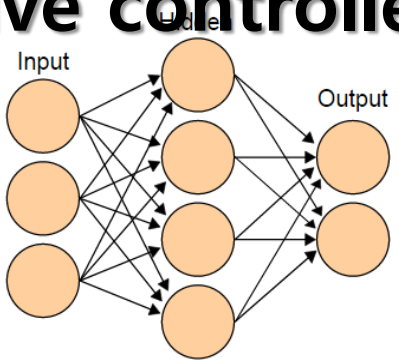
V. Future Plan.(1/1)

1. Prove the controller stability using Lyapunov Function.



2. Design the Controller Using Control Moment Gyros.(CMG)

3. Design the nonlinear adaptive controller using Neural Network.



Thank You~

CONTROLA

Lab. For Navigation, Control & Applications